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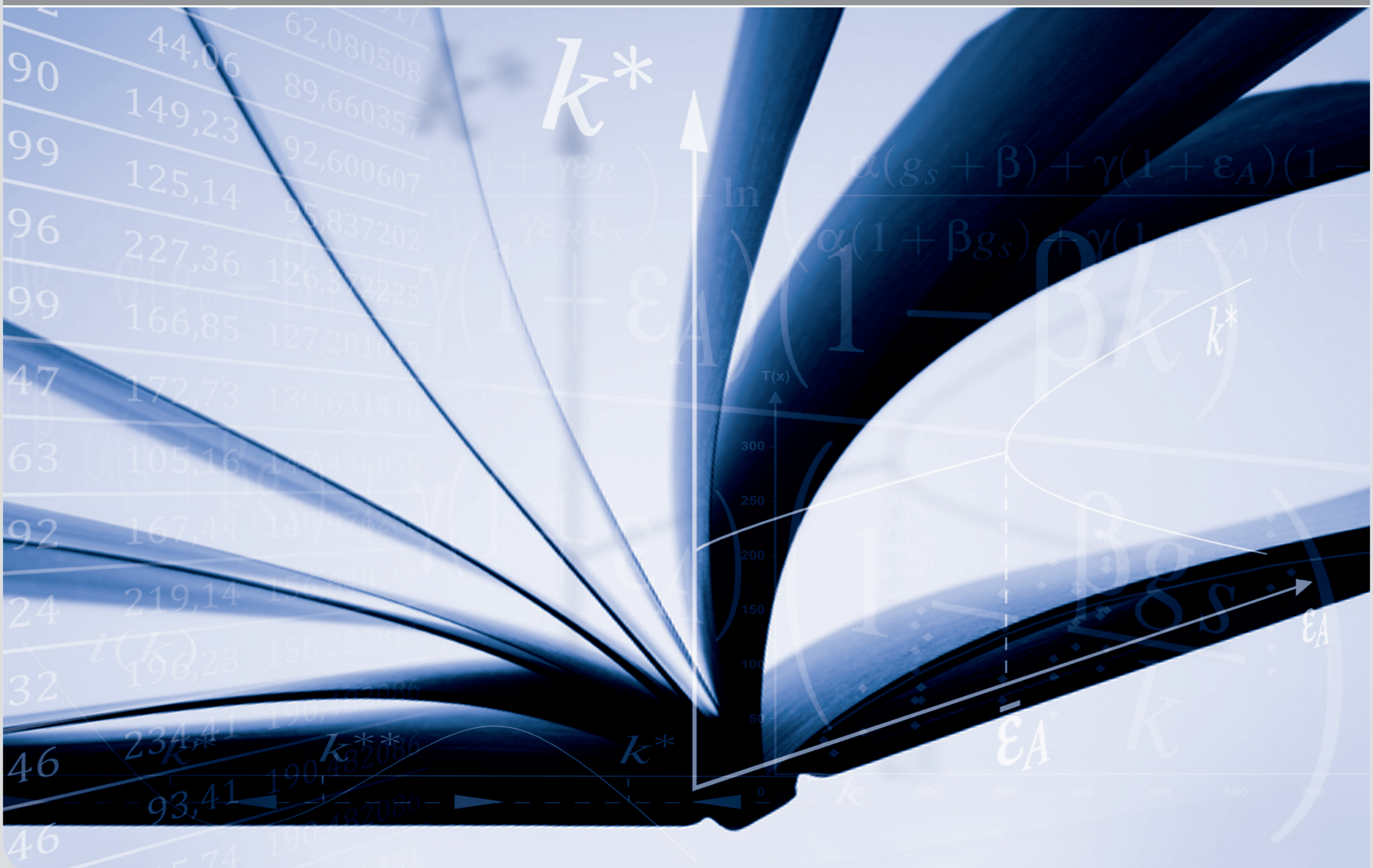

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Analysis of Stochastic Technical Trading Algorithms

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Abstract

We apply the well-known CUSUM, the Girshick-Rubin, the Graversen-Peskir-Shiryaev and an improved alteration of the Brodsky-Darkovsky algorithm as trading strategies involving only mutually exclusive long positions in cash and the DAX at Xetra intraday auction prices. We select optimal pairs of fixed thresholds for up- and downmovements from a pre-defined two-dimensional grid, hence, admitting asymmetric intervals. We show that under three different scenarios for transaction costs, the improved Brodsky-Darkovsky technique not only outperforms the passive investment in the DAX but also the other three presented algorithms.

Keywords: CUSUM, Girshick-Rubin, Graversen-Peskir-Shiryaev, Brodsky-Darkovsky, trading algorithm, DAX

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1. Introduction

One of the most critical questions in asset management and investing is the detection of changes in the current regime. The theoretical terminology refers to this as change-point detection or break-point analysis. In an economic context, models often involve a multitude of parameters, the stability of which over time has been put into question at least since Isaac and Griffin (1989). Many others such as, for example, Balding et al. (2008), Hamilton and Susmel (1994), Schaller and Van Norden (1997), Bai and Perron (1998), Hansen (2000), Dias and Embrechts (2002), Western and Kleykamp (2004) followed suit. These approaches detect change points by looking into the rearview mirror, that is, they analyze historical time series and determine the most probable scenario concerning a change in value, or multiple changes of a particular parameter of a more or less complex model, in the past. This, however, is of limited to no value to an investor or trader who has to receive signals immediately if a change appears likely.

Interestingly, a suitable approach has been provided by some technique developed for quality control in manufacturing, i.e. control chart techniques first developed such as Shewhart (1932). The general idea is to observe some time series until a predefined threshold is trespassed. Page (1954a) and Page (1954b) coined the term CUSUM as short for cumulative-sum where the actual value of a process, for example, a random walk is compared to some prior extreme value such as an all-time low or high, respectively. If the difference between actual and reference value is greater than the threshold, a signal is delivered. The initial approach was augmented by the moving average control chart by Roberts (2000).

The CUSUM is equivalent to the filter trading rule introduced by Alexander (1961). Initial results of this rule are given, for example, by Alexander (1961), Fama and Blume (1966), or Dryden (1969) who showed that, after consideration of trading costs, the filter method cannot outperform the traditional buy-and-hold strategy. Moreover, a shortcoming of the original rule was detected in that, under certain circumstances, the trading rule could result in unbounded losses.

In financial applications, methods of quickest detection of a change-point are of interest that are "free" of a distribution of a random sequence, i.e. nonparametric methods.[1] The biggest problem with control techniques is the proper determination of the threshold which my actually vary over time. Solutions in that context are provided by, for example, Verdier et al. (2008). It was shown that when the change-point is a random variable with known distribution, then the optimal method is to observe an à posteriori probability of a change-point until it reaches some threshold value which may be analytically calculated. This method, however, cannot be applied to problems arising in practice since it is almost always impossible to obtain any à priori information on the distribution of the time of occurrence of a change-point itself as well as on the distribution of a random sequence before and after the change. This, for example, makes the approach by Luo et al. (2009) who suggest variable sampling intervals under known distributions inapplicable, in our context. The most flexible approach so far has been suggested by Jeske et al. (2009) with, however, the still slightly unrealistic assumption of independent observations. A good overview of the topic is given by Wald (2013) and Shiryaev (2007).

[1]It turns out that it is possible to give "nonparametric" versions to some popular parametric methods.

2. Algorithms

Hereafter, the scheme of observations we deal with is as follows. Let $(\Omega, \mathcal{F}, P)$ be a probability space on which there is defined a random sequence $x = (x_i)_{i \geq 0}$ with

$$x_i = a + \xi_i \, I(i < i^*) + (c + \eta_i) \, I(i \geq i^*)$$

where $\xi = (\xi_i)_{i \geq 0}$ and $\eta = (\eta_i)_{i \geq 0}$ are random sequences such that $E(\xi_i) = E(\eta_i) = 0$ and a and c are constants with $a(a + c) < 0$. The index value i^* indicates a change-point.

The objective is to minimize the average delay until the detection of a true change-point while, at the same time, keeping the number of false alarms down. Among the algorithms we will present, there is no universal one for the quickest detection of a change-point in a variety of settings. Each one outperforms in its "domain".

Let

- initial value of time interval: T_0
- final value of time interval: T_1
- tick times of the asset price: $(\tau_j)_{j=0}^N$
- best bid price of the asset at tick: $(S_{\tau_j}^{bid})_{j=0}^N$
- best ask price of the asset at tick: $(S_{\tau_j}^{ask})_{j=0}^N$
- parameter of partition of time interval: Λ
- interval for smoothing: M

- coefficient of smoothing: $\alpha,\ 0 < \alpha < 1$
- threshold value of the algorithm: h

Let $L = [(T_1 - T_0)/\Lambda]$.[2] We define values of best bid and best ask asset prices at the points of (equidistant) partition $t_k = k\Lambda, (0 \le k \le L)$ of the time interval $[T_0, T_1]$ as values $S^{bid}_{\tau_m}$ and $S^{ask}_{\tau_m}$, respectively, at the tick times:

$$\tau_m = \max_{0 \le j \le N} \{\tau_j : \tau_j \le t_k\}.$$

In the following, by $S_k := S^{bid}_k - S^{ask}_k$, we will denote the mid-day auction prices with zero spread and the assumption of unlimited liquidity, instead. Also, we define

$$\xi_j = \ln S_j - \ln S_{j-1}, 1 \le j \le L.$$

2.1. CUSUM

Let k_0 be the last point in time that the signal (of a true change-point) was detected($k_0 = T_0$, at the beginning). Then, define recurrently

$$\begin{aligned} R^{(1)}_{k_0} &= R^{(2)}_{k_0} = 0 \\ R^{(1)}_k &= (R^{(1)}_{k-1} + \xi_k)^+ \\ R^{(2)}_k &= (-R^{(2)}_{k-1} + \xi_k)^-,\ k_0 < k \le L \end{aligned}$$

where we have used the convention $(a)^+ = \max\{a, 0\}$ and $(a)^- = -\min\{a, 0\}$. If $R^{(1)} \ge h$, for the first time since the last signal and for some pre-defined threshold $h > 0$, then, the algorithm indicates that the random sequence under observation shows an up-trend and, thus, sends a "buy" signal. If, on the other hand, $R^{(2)} \ge h$, then the random sequence shows a down-trend and the algorithm sends a "sell" signal.

[2] Here, $[a]$ stands for an integer part of a.

2.2. Girshick-Rubin

Let k_0 be the last point in time that a signal was detected. Define

$$\begin{aligned} R^{(1)}_{k_0} &= R^{(2)}_{k_0} = 0 \\ R^{(1)}_{k} &= 1/(k+1)\exp(\xi_k)(1 + kR^{(1)}_{k-1}) \\ R^{(2)}_{k} &= 1/(k+1)\exp(-\xi_k)(1 + kR^{(2)}_{k-1}),\ k_0 < k \le L \end{aligned}$$

If $R^{(1)} \ge h$, for some pre-defined threshold $h > 0$, then, there is a signal to buy the asset. On the other hand, if $R^{(2)} \ge h$, then there is a signal to sell the asset.

2.3. Graversen-Peskir-Shiryaev

Let k_0 be the last point in time that a signal was detected. Define

$$\begin{aligned} R^{(1)}_{k} &= S_k - \max_{k_0 \le i \le k-1} S_i \\ R^{(2)}_{k} &= \max_{k_0 \le i \le k-1} S_i - S_k,\ k_0 < k \le L \end{aligned}$$

The algorithm indicates an up-trend if $R^{(1)} \ge h$ for some pre-defined threshold $h > 0$. In this case there is a signal to buy the asset. However, there is a signal to sell the asset if $R^{(2)}_k \ge h$, which indicates a down-trend.

2.4. Brodsky-Darkovsky

First we need to introduce some additional parameters:

- K: parameter of time frame
- $(\beta_k)_{k=1,2}$: parameter of truncation $(0 < \beta_1 < \beta_2 < 1)$

Let k_0 be the last point in time that a signal was detected. Here we assume that $k_0 > K$. Define

$$Y(m,l) = \frac{1}{m}\sum_{k=l-K+1}^{l-K+m} \xi_k - \frac{1}{K-m}\sum_{k=l-K+m+1}^{l} \xi_k,\ l = k_0, k_0+1, \ldots.$$

If $\min_{[\beta_1 K] \leq m \leq [\beta_2 K]} Y(m,l) \leq -h$ for some pre-defined threshold $h > 0$, then there is a signal to buy the asset. If $\max_{[\beta_1 K] \leq m \leq [\beta_2 K]} Y(m,l) \geq h$, then there is a signal to sell the asset.

2.5. Modified Brodsky-Darkovsky

Let the set be the same as in the previous presented Brodsky-Darkovsky Algorithm. Define

$$Y(m,l) = \frac{1}{m} \sum_{i=l-K+1}^{l-K+m} \xi_i - \frac{1}{K-m} \sum_{i=l-K+m+1}^{l} \xi_i, \; l = k_0, k_0+1, \ldots .$$

Let us consider the following case analysis and introduce two other parameters:

- m_{min} for m that fullfills $\underset{[\beta_1 K] \leq m \leq [\beta_2 K]}{min} Y(m,l) \leq -h$
- m_{max} for m that fullfills $\underset{[\beta_1 K] \leq m \leq [\beta_2 K]}{max} Y(m,l) \geq h$

Define

$$D(m,l) = \frac{1}{K-m} \sum_{i=l-K+m+1}^{l} \xi_i - \frac{1}{\lfloor \frac{K-m}{2} \rfloor} \sum_{i=l-K+m+\lfloor \frac{K-m}{2} \rfloor+1}^{l} \xi_i$$

If $D(m,l) > h$, where $m = m_{min}$, than there is a signal to buy the asset.
If $D(m,l) < h$, where $m = m_{max}$, than there is a signal to sell the asset.

2.6. Concluding remarks

At this point, it ought to be mentioned that the threshold value of the algorithm h as far as the other parameters Λ, a, and c should be determined by trial-and-error methods or grid search. For simplicity, we only assume h to be variable and keep the other parameters fixed ($a = -1$, $c = +2$, and $\Lambda = 1$).[3] To provide for

[3] As of yet, we do not (!) have a preferred algorithm for the determination of these parameters.

greater flexibility, we allow h for up-movements to be different from h for down-movements. The modified Brodsky-Darkovsky algorithm is an advancement of the simple Brodsky-Darkovsky and focuses on the analysis of only few most latest input returns, what enables more precise results. However the empirical study of the modified Brodsky-Darkovsky algorithm is not discussed in this paper.

3. Data

We obtained the daily Xetra one-minute intraday auction prices of the DAX index (WKN 846900) from the Karlsruher "Kapitalmarktdatenbank". Our sample covers the period from January 3, 2000 to December 30, 2013, yielding over 1.8 Million observations. Thus, it contains observations from two different crises: the dot-com bubble in 2000 and the latest crisis beginning 2007. Figure 1 displays a chart of the DAX level in that period. The currency is euros, implying a backward conversion of the prices in Deutsche Mark prior to January 1, 2002. Starting at a level of EUR 6976.12, the DAX gained EUR 2576.04 over the sample period resulting in a level of EUR 9552.16 at the end of December, 2013.

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Figure 1 here

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In Figure 2, we display the log-returns of the DAX over the sample period. The minimum is -0.0735 and the maximum 0.0533 with a mean of nearly zero. The standard deviation is 0.0007 while skewness and kurtosis are -0.0612 and 375.4088, respectively. This hints at a strongly leptocurtic and asymmetric distribution of the log-returns which is supported by the very high Jarque-Bera test statistic of 10449851687. The 0.05- and 0.95-quantiles are equal to -0.0008 and 0.0008, respectively. A kernel density plot is given by Figure 3.

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Figure 2 here

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Figure 3 here

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4. Set-up and Results

4.1. General Set-up

Our approach is as follows: we start with an initial cash position of 50,000 euros at the beginning, on January 3, 2000. At the end, on December 30, 2013, we dissolve any investment at current value if a long position is taken; otherwise, we simply consider the cash position. In between, we either invest the entire current amount at the current price of the DAX (if a buy signal occurs) and hold it until the next sell signal or sell everything and hold cash only (if a sell-signal occurs). Thus, portfolio weights for cash and asset are mutually exclusively either zero or one. The positions are held, respectively, until the first signal of opposite sign is observed. Hence, consecutive signals of same sign do not lead to any action.

Further, we assume three scenarios. In the first scenario, there are no transaction costs, so we can alter positions at zero expense. Under the second scenario, transactions are assumed to cost EUR 5 for both sell and buy. The third scenario has transaction fees of EUR 30 for both sell and buy. Thereby the transaction costs are assumed as fixed and independent of trading volume.[4].

In this paper, the transaction is executed on the subsequent auction price traded on the term coming directly after the trading signal. The optimal solution for the threshold parameter h is selected per grid search. To provide greater flexibility we allow the bounds to be assymetrical, that is, the threshold h for up-movements (which we denote as h_1 here) can be different from the threshold for down-movements (which we denote as h_2 here).[5] With respect to computer software, we used Java and R.

4.2. CUSUM Results

To get an overview of the best area of the optimal threshold in the CUSUM strategy, we use two different grids. The coarser one covers the domain G_1 =

[4]We admit that this may be somewhat unrealistic especially for the earlier years of the sample period. The approximate transaction costs of EUR 5 is provided by some online banks (Flatex, 2015) and EUR 30 can be observed at the most branch banks. Also, we assume unlimited liquidity on both the sell and buy side to hold prices stable. We concede that this might leave room for improvement. However, we do not think that this will impair the overall picture obtained from our analysis.

[5]One should keep in mind that symmetrical bounds translate into asymmetrical bounds for actual DAX level changes since we focus on log-returns.

$[0.01, 0.1]$ with step size 0.01. The other grid covers the domain $G_2 = [0.1, 1.5]$ with step size 0.1. Since absolute changes within the grid G_1 represents a larger relative step than in grid G_2, the step size in the first grid is smaller than in the second grid.

First, we discuss the results on G_1. The bounds within this grid are so small, that trading signals are activated yet for small differences in the price development and therefore more frequent compared to greater thresholds. Since the gap between the buy and the sell price is small, the introduction of fees destroys small profits and leads to a drastic reduction of the final pay-off. For the third scenario of EUR 30 fees the strategy almost completely destroys the trading capital.

In comparison to G_1 the analysis of the second grid G_2 ends in a greater final pay-off. The results within G_2 are more stable with respect to the introduction of fees, since the number of trading signals is much smaller than for smaller thresholds. Moreover the profits are greater due to bigger differences between the buy and sell prices.

The refinement of the grid around the most promising location in G_2 results into the best parameter choice $h_1 = 0.278$ and $h_2 = 0.196$ with the optimal pay-off of 172602.31 euros under the first scenario and 171801.24 euros under scenario three. As can be seen in Figure 6 this outcome is stable around the location with the highest payoff.

4.3. Girshick-Rubin Results

As in the CUSUM strategy we here use the same grids G_1 and G_2. However, in contrast to CUSUM this method is based on absolute returns, which are greater than logarithmic returns. Therefore, less trading signals in the area of small thresholds occur and the introduction of fees has a relatively small impact on the final pay-off.

Both highest pay-offs in the grids G_1 and G_2 are not stable and result under the impact of random deviation (see Figure 9). Nevertheless the bounds $h_1 = 0.078$ and $h_2 = 0.076$ lead to the highest final pay-off of 174252.25 euros under scenario one and 171229.66 euros under scenario three.

4.4. Graversen-Peskir-Shiryaev Results

Unlike the first two algorithms, this method evaluates absolute DAX values. As already presented, this method yields a buy signal if $S_k - \max_{k_0 \le i \le k-1} S_i \ge h_1$. Hence, the difference between the maximum and the current observed DAX value cannot exceed the maximum increase in DAX points within one period. Therefore, we set the highest possible h_1 on the maximum increase in DAX points. However the down-trend covers all possible differences between the DAX values. For this reason we set h_2 on the difference between the highest and lowest DAX value. Thus, we set up the grid on $h_1 = [1, 246]$ with step size 3.5 and $h_2 = [1, 7001]$ with step size 100. As can be seen in Figure 10 there are two areas with higher results. We are interested in the area with smaller thresholds since the area with h_2 around 4000 represents the Buy-and-Hold strategy. The refinement of the grid around the most promising location achieves under scenario one the maximum final pay-off of 156013.3 for $h_1 = 98$ and $h_2 = 1985$. Even though this profit does not reduce much under scenario three and is 155620.30 euros, the results are not stable and result in losses for the most part after the introduction of fees.

4.5. Brodsky-Darkovsky Results

Since the Brodsky-Darkovsky strategy contains more parameters than the first three methods, it enables a deeper data analysis. As we are unaware of the optimal areas for parameter values, we first analyse a common coarse grid for all three scenarios. Therefore we use for thresolds h_1 and h_2 the grid $[0.0001, 0.1001]$ with step size 0.02 and for the parameter K the grid [18, 61] with step size 7. As presented in the section above, β_1 and β_2 determine how to split the last K data points into two sections in order to calculate their respective means. To avoid a distortion of results, we allow each section to contain at least four values. Thus we use for β_1 the grid $[\frac{4}{K}, 0.5 + \frac{5}{K}]$ with step size 0.1 and for β_2 the grid $[0.6 - \frac{4}{K}, \frac{K-4}{K}]$ with step size 0.1. From the resulting optimal solution for the parameters we can observe, that increasing fees lead to greater thresholds h_1 and h_2, greater K and β_2 and to smaller β_1. According to these observations, we refine the grid around the most promising locations for parameters and achieve an increase from approximately 1.2 Mio euros to approximately 2.4 Mio euros under scenario one. The resulting optimal choice for the parameters is $h1 = h2 = 0.00005$, $\beta_1 = 0.35$, $\beta_2 = 0.45$ and $K = 19$. Even under scenario three, this method leads to higher final pay-offs than the other three strategies and results in final pay-off of 321901.96 euros. The according optimal parameters are $h_1 = 0.0096$, $h_2 = 0.0112$, $\beta_1 = 0.08$, $\beta_2 = 0.9$ and $K = 53$.

5. Summary

We have presented four different trading algorithms, the well-known CUSUM technique, as well as the Girshick-Rubin, the Graversen-Peskir-Shiryaev and the Brodsky-Darkovsky method. All Algorithms were tested on minute-by-minute DAX valuations over a long period of time and the optimal set of possibly asymmetric parameters was selected from a multi-dimensional grid. While we admit that we make some naïve assumptions concerning unlimited liquidity at the Xetra end-of-day auction ("Kassakurs”), we provide for some realistic constraints by introducing several scenarios for trading fees.

While for the CUSUM algorithm we observe stable pay-offs for optimal thresholds around 0.2, neither for Girshick-Rubin nor for Graversen-Peskir-Shiryaev, stable areas with high pay-offs could be found. Moreover, in some of the realistic scenarios (i.e. incorporating transaction costs), the last two algorithms led to ruin. Overall, we observe that the Brodsky-Darkovsky algorithm yields superior results over the other presented algorithms. This can be explained by the limitation of input data to only K observations for the analysis of each return. By taking only the K last log-returns into consideration we use data that have the higher effect on the current observation. Additionaly, this limitation reduces the probability of the distortion of results due to the long-term effect of outliers. This leads to greater importance of each individual log-return and achieves higher final pay-offs. We noticed that the higher the fees, the greater the optimal thresholds, the K and β_2 value and the smaller is β_1.

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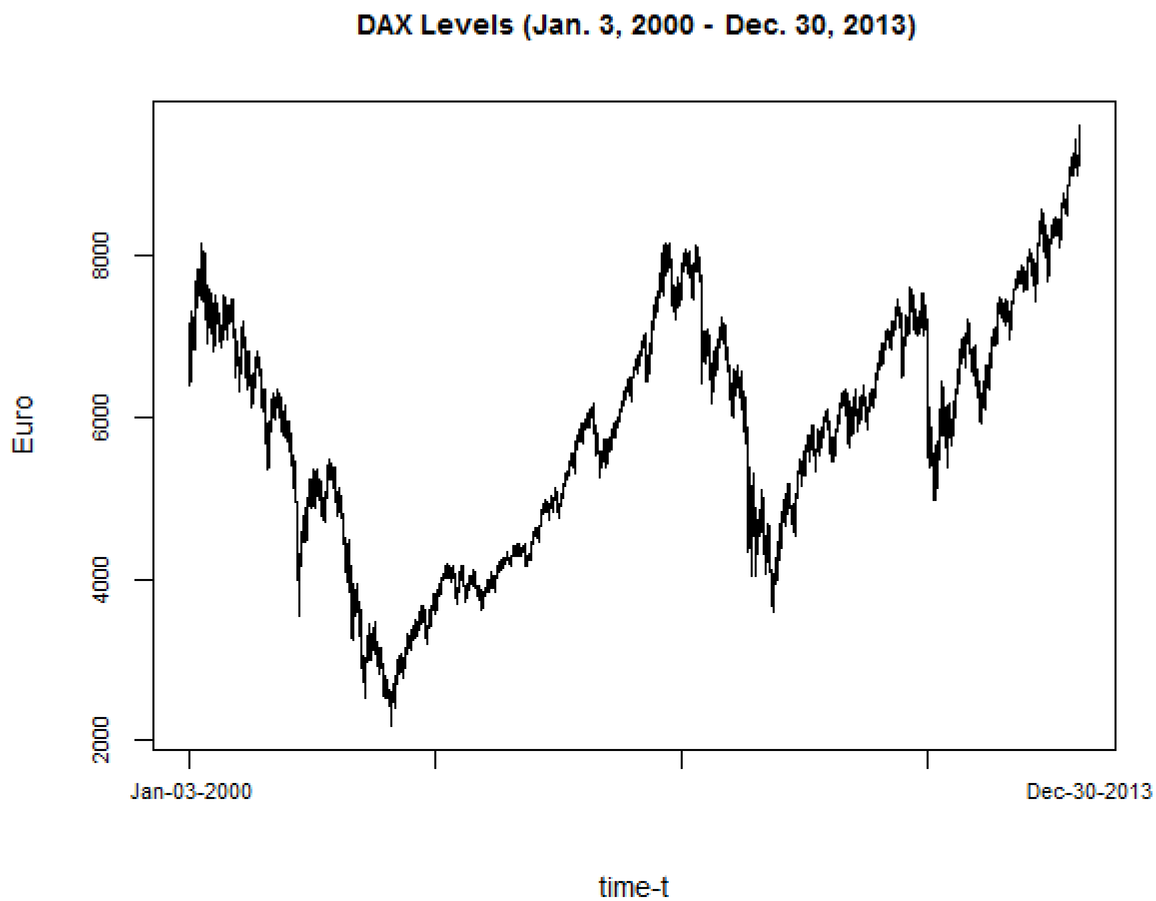


Figure 1: DAX levels

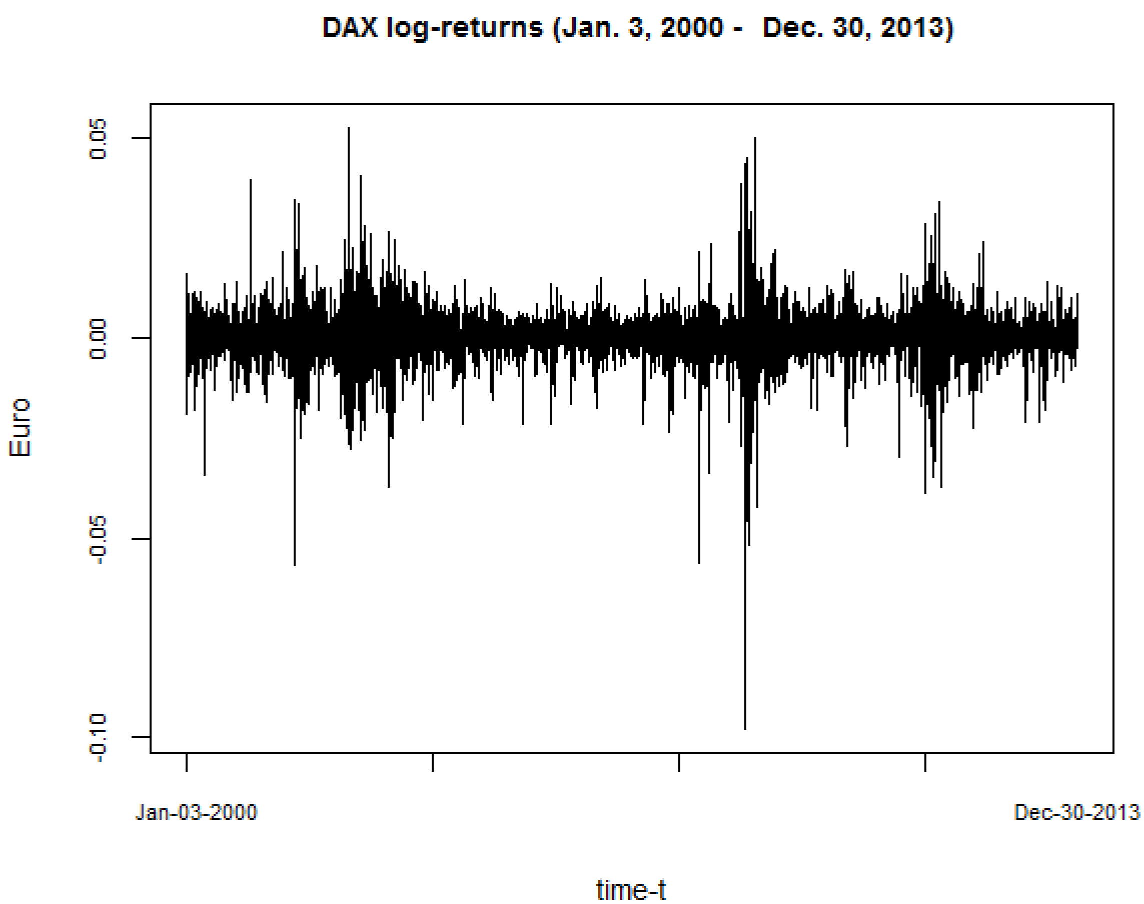


Figure 2: DAX log-returns

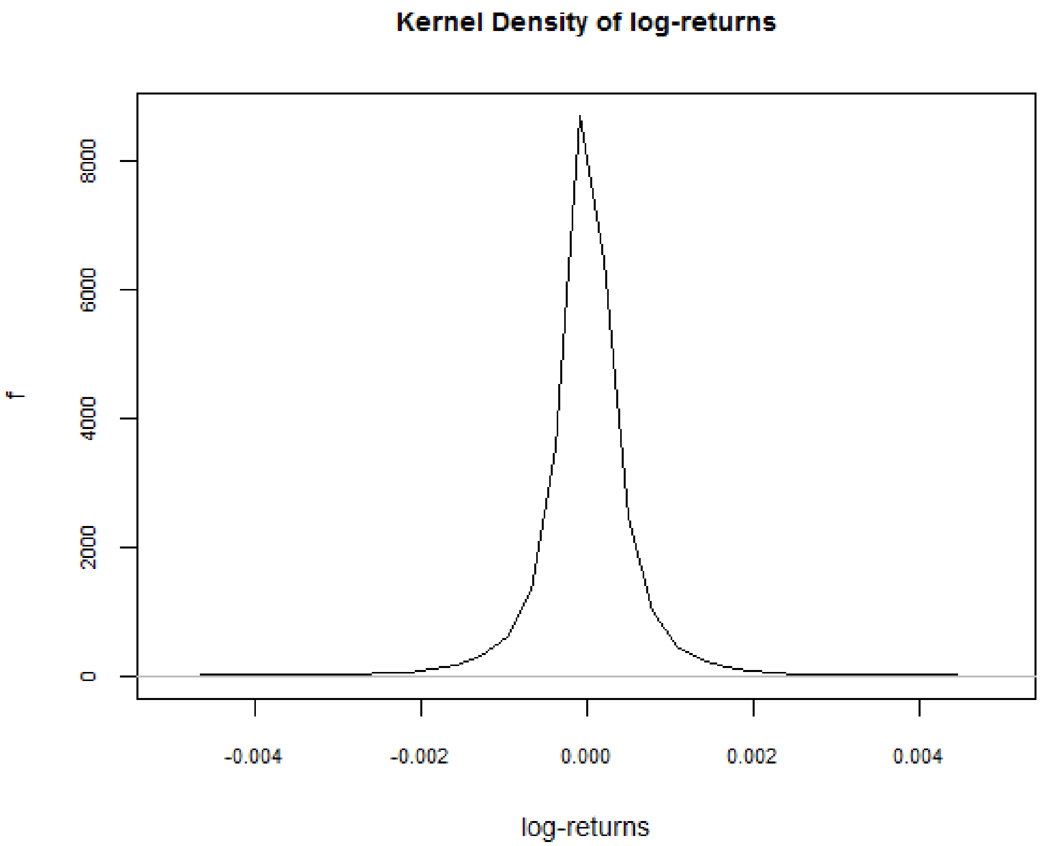
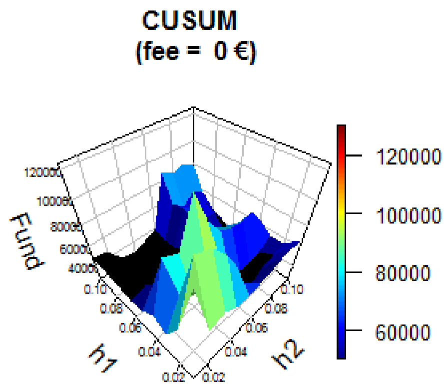


Figure 3: kernel density estimation



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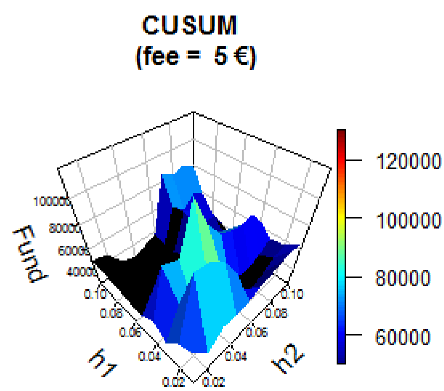

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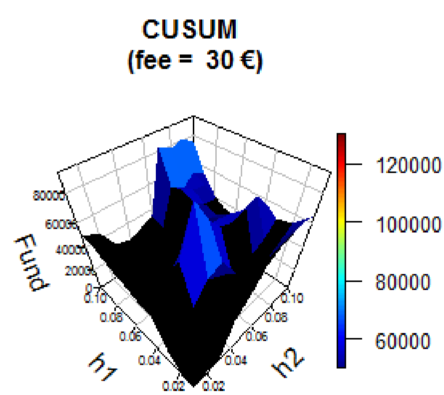
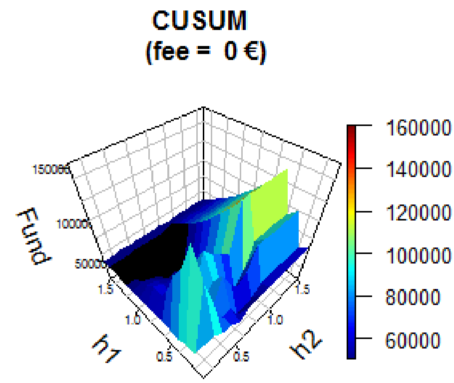


(c)

Figure 4: Comparison of CUSUM strategies for different fees ($h_1, h_2 \in \{0.01, 0.02, \dots, 0.1\}$).



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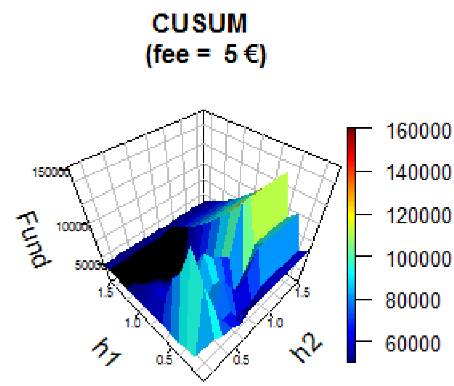

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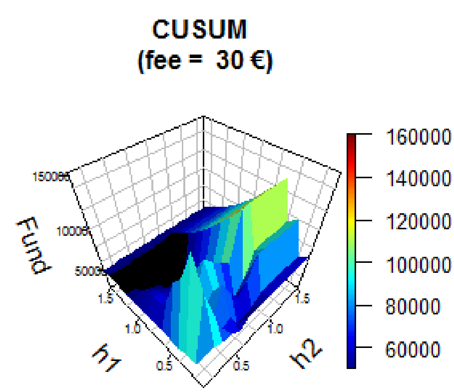
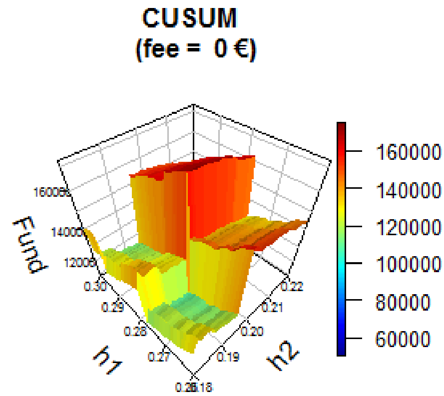
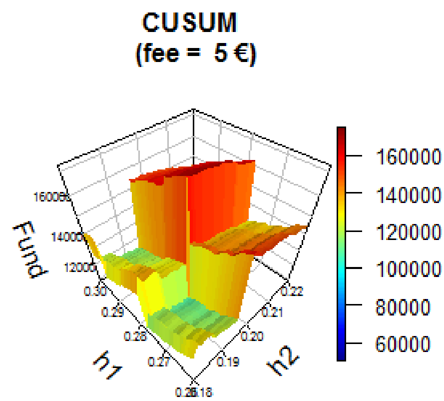


(c)

Figure 5: Comparison of CUSUM strategies for different fees ($h_1, h_2 \in \{0.1, 0.2, \ldots, 1.5\}$).



(a)



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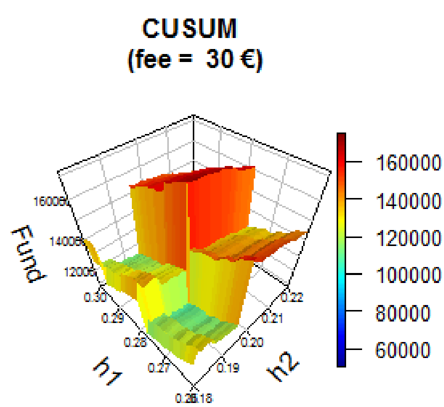
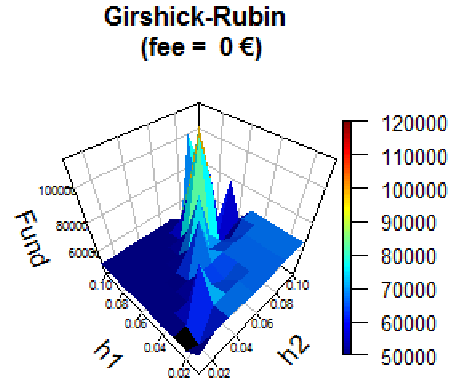
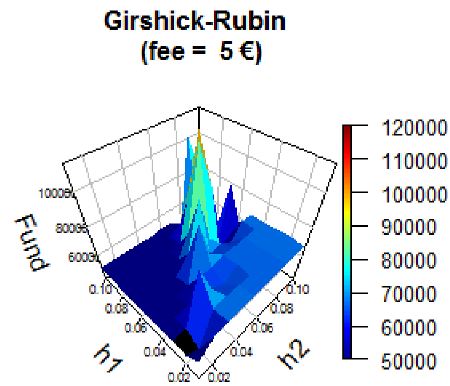


(c)

Figure 6: Comparison of CUSUM strategies for different fees ($h_1 \in \{0.26, 0.261, \ldots, 0.3\}$, $h_2 \in \{0.18, 0.181, \ldots, 0.22\}$).



(a)



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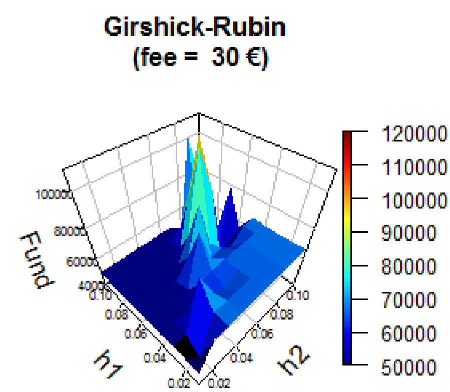
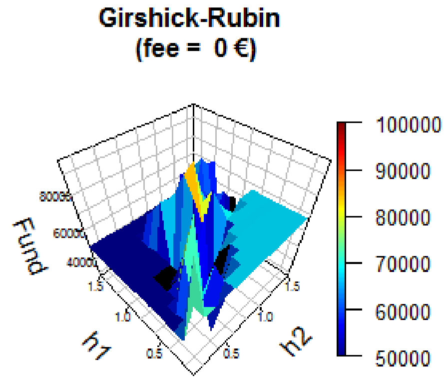
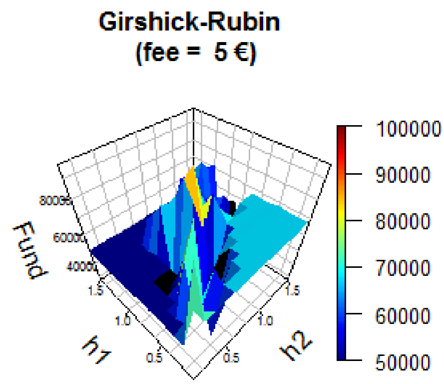


(c)

Figure 7: Comparison of Girshick-Rubin strategies for different fees ($h_1, h_2 \in \{0.01, 0.02, \ldots, 0.1\}$).



(a)



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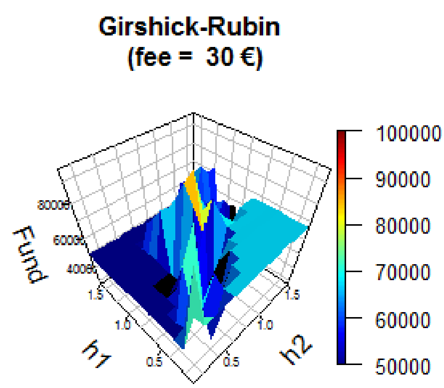
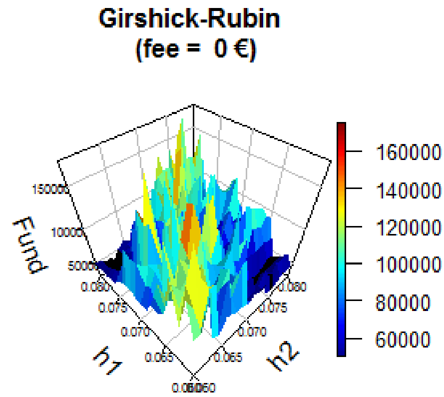


(c)

Figure 8: Comparison of GR strategies for different fees ($h_1, h_2 \in \{0.1, 0.2, \ldots, 1.5\}$).



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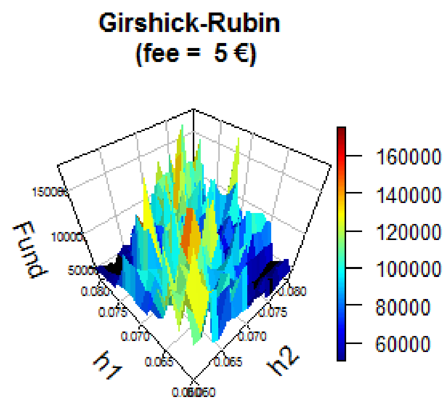

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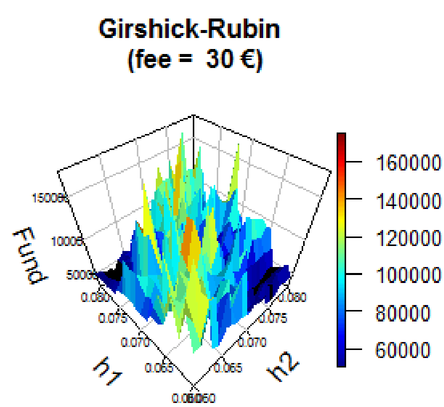
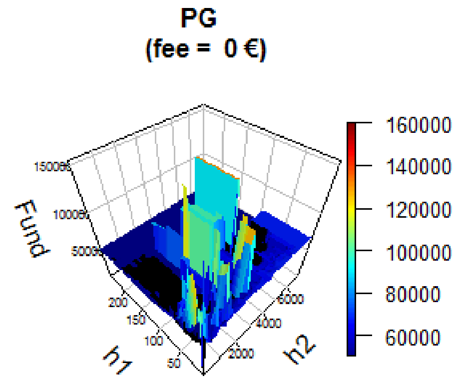
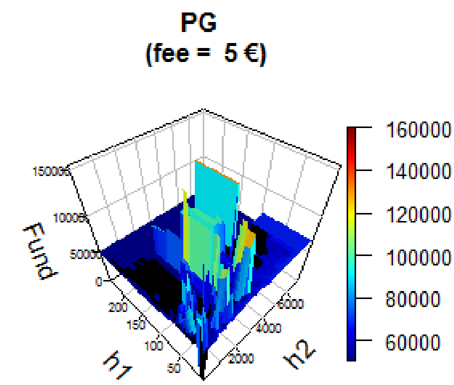


(c)

Figure 9: Comparison of Girshick-Rubin strategies for different fees ($h_1, h_2 \in \{0.06, 0.061, \ldots, 0.08\}$).



(a)



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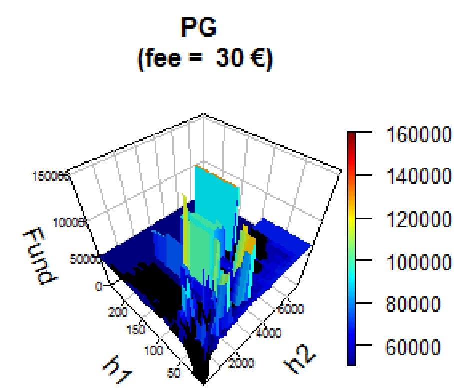
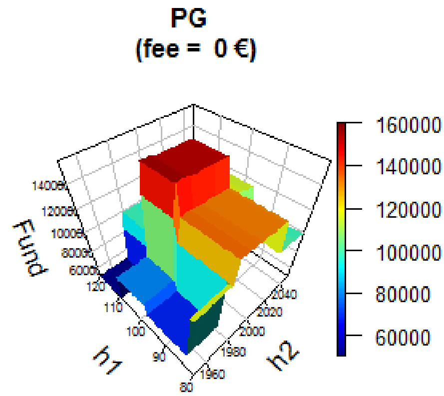
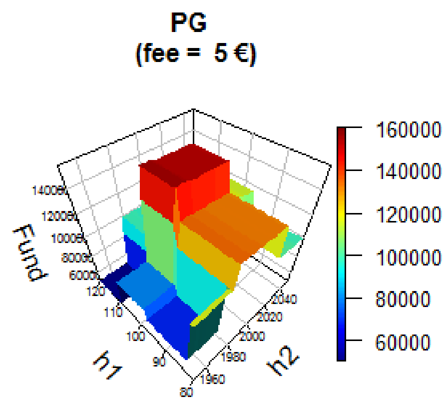


(c)

Figure 10: Comparison of Peskir-Graversen strategies for different fees ($h_1 \in \{1, 4.5 \ldots, 246\}$, $h_2 \in \{1, 101 \ldots, 7001\}$).



(a)



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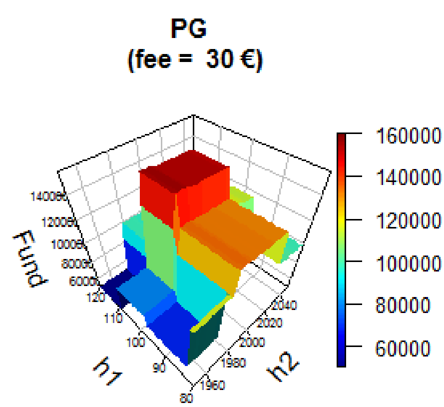
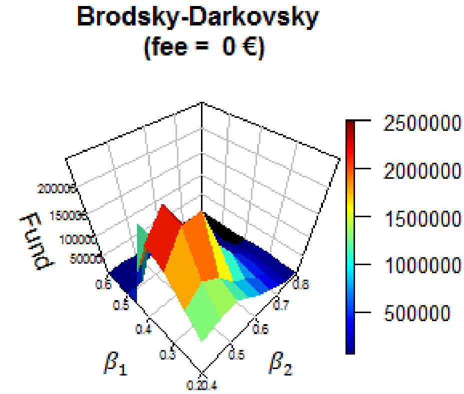
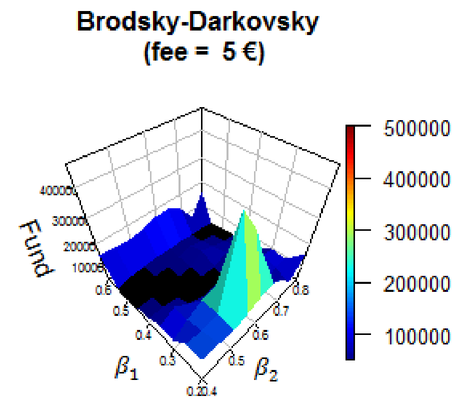


(c)

Figure 11: Comparison of Peskir-Graversen strategies for different fees ($h_1 \in \{80, 81, \dots, 120\}$, $h_2 \in \{1950, 1952.5, \dots, 2050\}$).



(a)



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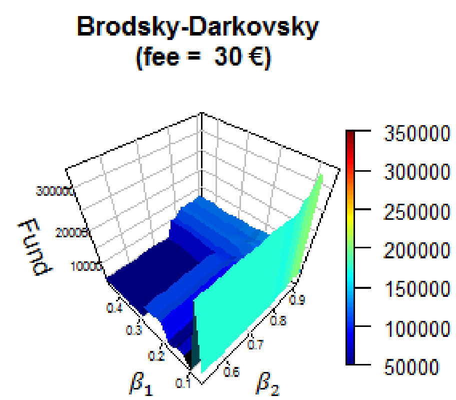
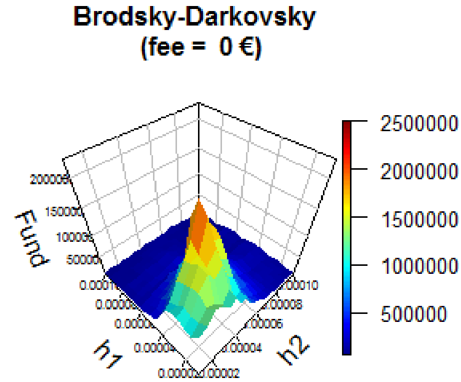
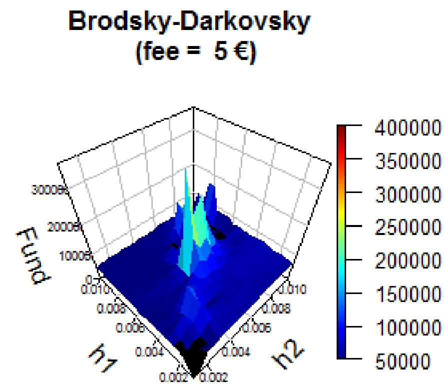


(c)

Figure 12: Comparison of Brodsky-Darkovsky strategies for different fees depending on β_1 and β_2 ($\beta_1 \in [0.2, 0.25, \ldots, 0.6], \beta_2 \in [0.4, 0.6, \ldots, 0.8]$ for $h1 = 0.00005,\ h2 = 0.00005,\ K = 19$ (a), $\beta_1 \in [0.2, 0.25, \ldots, 0.6], \beta_2 \in [0.4, 0.6, \ldots, 0.8]$ for $h1 = 0.0056,\ h2 = 0.0052,\ K = 24$ (b) and $\beta_1 \in [0.06, 0.08, \ldots, 0.46], \beta_2 \in [0.52, 0.54, \ldots, 0.92]$ for $h1 = 0.0096,\ h2 = 0.0112,\ K = 53$ (c)).



(a)



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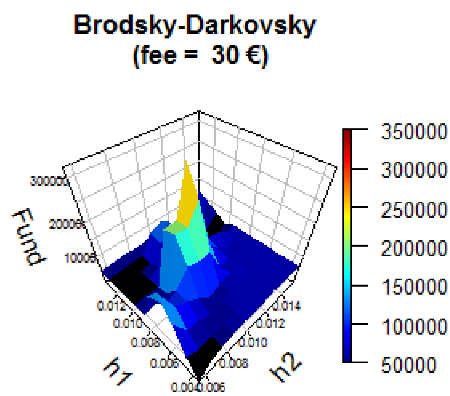

(c)

Figure 13: Comparison of Brodsky-Darkovsky strategies for different fees depending on h_1 and h_2 ($h_1, h_2 \in [0.00001, 0.000015, \ldots, 0.0001]$ for $\beta_1 = 0.35, \beta_2 = 0.45$, $K = 19$ (a), $h1, h2 \in [0.001, 0.0015, \ldots, 0.01]$ for $\beta_1 = 0.25$, $\beta_2 = 0.6$, $K = 24$ (b) and $h_1 \in [0.0035, 0.0045, \ldots, 0.0135], h_2 \in [0.0055, 0.0065, \ldots, 0.0155]$ for $K = 53, \beta_1 = 0.08, \beta_2 = 0.9$ (c))

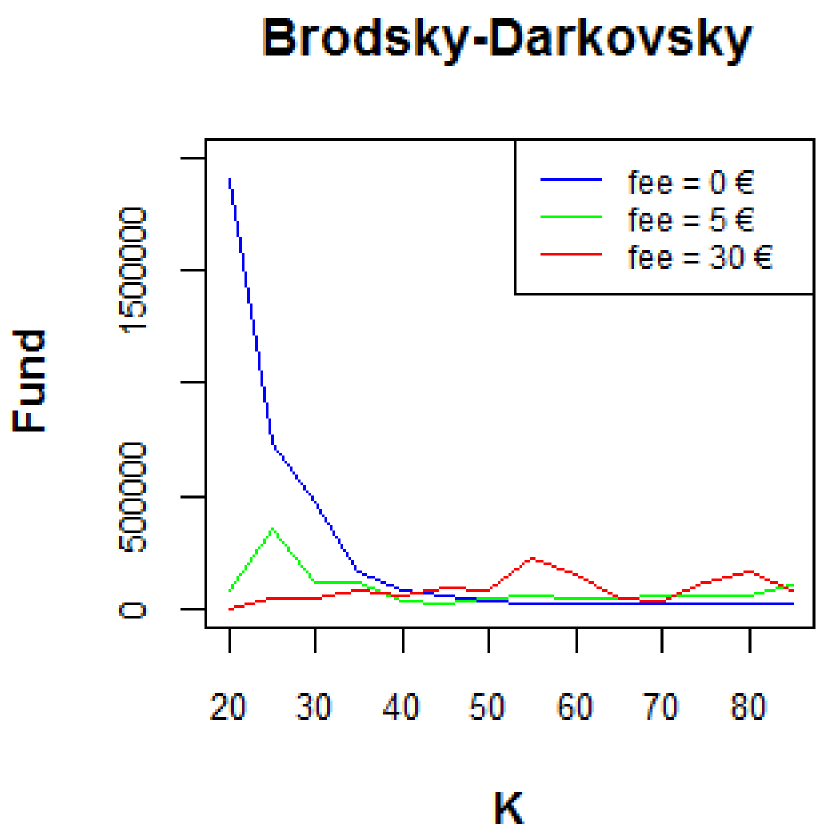

Figure 14: Comparison of Brodsky-Darkovsky strategies for different fees depending on K

Algorithm	**Fee**	**Grid**	$h1^*$	h_2^*	β_1^*	β_2^*	K^*	**Pay-off**
CUSUM	0	$h1, h2 \in [0.1, 0.2, \dots, 2]$	0.3	0.2				153296.64 e
	5	$h1, h2 \in [0.1, 0.2, \dots, 2]$	0.3	0.2				153171.96 e
	30	$h1, h2 \in [0.1, 0.2, \dots, 2]$	0.3	0.2				152548.52 e
Girshick-Rubin	0	$h1, h2 \in [0.01, 0.02, \dots, 0.1]$	0.07	0.07				113866.30 e
	5	$h1, h2 \in [0.01, 0.02, \dots, 0.1]$	0.07	0.07				113206.30 e
	30	$h1, h2 \in [0.01, 0.02, \dots, 0.1]$	0.07	0.07				109906.72 e
Graversen-Peskir-Shiryaev	0	$h1 \in [1, 4.5 \dots, 246]$, $h2 \in [1, 101, \dots, 7001]$	99	2001				154317.56 e
	5	$h1 \in [1, 4.5 \dots, 246]$, $h2 \in [1, 101, \dots, 7001]$	99	2001				154252.60 e
	30	$h1 \in [1, 4.5 \dots, 246]$, $h2 \in [1, 101, \dots, 7001]$	99	2001				153927.73 e
Brodsky-Darkovsky	0	$h1, h2 \in [0.0001, 0.0201 \dots, 0.1001]$, $\beta_1 \in [\frac{4}{K}, \frac{4}{K} + 0.1, \dots, 0.5 + \frac{4}{K}]$, $\beta_2 \in [0.6 - \frac{4}{K}, 0.7 - \frac{4}{K}, \dots, \frac{(K-4)}{K}]$, $K \in [18, 25, \dots, 61]$	0.0001	0.0001	0.32	0.38	18	1220291.14 e
	5	$h1, h2 \in [0.0001, 0.0201 \dots, 0.1001]$, $\beta_1 \in [\frac{4}{K}, \frac{4}{K} + 0.1, \dots, 0.5 + \frac{4}{K}]$, $\beta_2 \in [0.6 - \frac{4}{K}, 0.7 - \frac{4}{K}, \dots, \frac{(K-4)}{K}]$, $K \in [18, 25, \dots, 61]$	0.0051	0.0051	0.26	0.6	25	366435.42 e
	30	$h1, h2 \in [0.0001, 0.0051 \dots, 0.1001]$, $\beta_1 \in [\frac{4}{K}, \frac{4}{K} + 0.1, \dots, 0.5 + \frac{4}{K}]$, $\beta_2 \in [0.6 - \frac{4}{K}, 0.7 - \frac{4}{K}, \dots, \frac{(K-4)}{K}]$, $K \in [18, 25, \dots, 61]$	0.0101	0.0101	0.09	0.9	46	112350.43 e

Table 1: Final optimal pay-off per strategy and coarsely grid

Algorithm	**Fee**	**Grid**	$h1^*$	h_2^*	β_1^*	β_2^*	K^*	**Pay-off**
CUSUM	0	$h1 \in [0.26, 0.261, \dots, 0.3]$, $h2 \in [0.18, 0.181, \dots, 0.22]$	0.278	0.196				172602.31 e
	5	$h1 \in [0.26, 0.261, \dots, 0.3]$, $h2 \in [0.18, 0.181, \dots, 0.22]$	0.278	0.196				172468.80 e
	30	$h1 \in [0.26, 0.261, \dots, 0.3]$, $h2 \in [0.18, 0.181, \dots, 0.22]$	0.278	0.196				171801.24 e
Girshick-Rubin	0	$h1, h2 \in [0.06, 0.061, \dots, 0.08]$	0.078	0.076				174252.25 e
	5	$h1, h2 \in [0.06, 0.061, \dots, 0.08]$	0.078	0.076				173748.48 e
	30	$h1, h2 \in [0.06, 0.061, \dots, 0.08]$	0.078	0.076				171229.66 e
Graversen-Peskir-Shiryaev	0	$h1 \in [80, 81, \dots, 120]$, $h2 \in [1950, 1952.5, \dots, 2050]$	98	1985				156013.35 e
	5	$h1 \in [80, 81, \dots, 120]$, $h2 \in [1950, 1952.5, \dots, 2050]$	98	1985				155947.84 e
	30	$h1 \in [80, 81, \dots, 120]$, $h2 \in [1950, 1952.5, \dots, 2050]$	98	1985				155620.30 e
Brodsky-Darkovsky	0	$h1, h2 \in [0.00001, 0.00002 \dots, 0.00008]$, $\beta_1 \in [0.2, 0.25, \dots, 0.4]$, $\beta_2 \in [0.25, 0.3, \dots, 0.45$, $K \in [18, 19, \dots, 22]$	0.00005	0.00005	0.35	0.45	19	2446054.09 e
	5	$h1, h2 \in [0.0048, 0.0050 \dots, 0.006]$, $\beta_1 \in [0.2, 0.25, 0.3]$, $\beta_2 \in [0.55, 0.6, 0.65]$, $K \in [22, 23, \dots, 28]$	0.0056	0.0052	0.25	0.6	24	468768.69 e
	30	$h1, h2 \in [0.008, 0.0088 \dots, 0.0112]$, $\beta_1 \in [\frac{4}{K}, \frac{4}{K} + 0.05, \dots, 0.13]$, $\beta_2 \in [0.8, 0.85, \dots, \frac{K-4}{K}]$, $K \in [50, 51, \dots, 56]$	0.0096	0.0112	0.08	0.9	53	321901.96 e

Table 2: Final optimal pay-off per strategy and fine grid

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