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**Decision making in politics and economics:
4. Bundestag elections 2009 and direct democracy**

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Abstract

Five German leading parties and their coalitions are evaluated from the viewpoint of direct democracy. For this purpose, the positions of the parties on over 30 topical issues are compared with the results of polls of public opinion. The outcomes are summarized in the indices of popularity and universality of the parties and of the DGB (German Confederation of Trade Unions). The selection of policy issues and the information on the party positions are given as in the Wahl-O-Mat(2010) for the last Bundestag (German parliamentary) elections 2009.

It is shown that the Bundestag election winner 2009 — the conservative party CDU/CSU with 33.8% votes — has a quite low representative capacity (fourth among the five leading parties), whereas the most representative is the left party *Linke* which received only 11.9% votes. As for possible coalitions, the most representative would consist of the *Linke* and the ecologists *Grünen*, who received together 22.6% votes and could not make a government. It is noteworthy that the DGB is also top evaluated as a good representative of majority opinions.

The analysis of Bundestag elections 2009 shows that the voters are little consistent with their own political profiles, disregard party manifestos, and are likely driven by political traditions, even if outdated, or by personal images of politicians. A possible explanation is that the spectrum of the German political landscape has significantly shifted to the right, whereas voters still believe that the parties represent the same values as a few decades ago. The insufficient focus on the changing party profiles results in a discrepancy between electorate and elected government. In case of Germany it manifests itself in the unprecedented violent response of population to some rather usual events like starting the construction of a new main station in Stuttgart ('Stuttgart-21'), or transporting atomic waste ('Castor Transport').

Taking into account the results of the study, some modifications to the election procedure are proposed to bridge approaches of representative and direct democracy. It is suggested to introduce the third vote in the form of 'sample referenda' with voters' Yes/No opinions on several important issues from party manifestos. It meets the existing logic of the German two-vote system: the first vote for a person, the second vote for a party, and the third vote for party profiles, so that the considerations are getting to be more conceptual and less personified.

Keywords: Representative democracy, direct democracy, elections, coalitions, theory of voting, mathematical theory of democracy, indices of popularity and universality.

JEL Classification: D71

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1 Introduction

The difference between direct democracy and representative democracy is in the way sovereignty is exercised — by the assembly of all citizens, or by elected representatives. Direct democracy, called also pure democracy, is generally regarded as the most advanced form of democracy. Respectively, representative democracies sometimes practice elements of direct democracy — referenda (plebiscites) — on most important political issues.

Shortcomings of representative democracy are caused by some particularities of voting, and by intermediation of voters' will by representatives. For instance, the bottle-necks of simple majority voting are so critical that the legitimacy of election results can be put in question (Held 1996, Samons 2004). As noticed by Borda as early as in 1770, if no candidate gets an absolute majority then the election winner can be most undesirable for an absolute majority. He illustrated it with an example of 21 voters with the rankings of three candidates A, B, and C shown in Figure 1 (Black 1958, p. 157). Indeed, the election winner candidate A with 8 votes is the most undesirable for 13 voters of 21.

Rankings	$A \uparrow$ $ $ $B \uparrow$ $ $ $C \uparrow$ $ $	$B \uparrow$ $ $ $C \uparrow$ $ $ $A \uparrow$ $ $	$C \uparrow$ $ $ $B \uparrow$ $ $ $A \uparrow$ $ $
Voters	8	7	6

Figure 1: Borda's example of 1770 as given by Black (1958)

In similar cases, more information than just the first choices should be considered: candidate rankings (preferences of electors with second and third priorities), preference grades, quantitative estimations, etc. However, these methods either have questionable assumptions, or can result in cyclic orders of candidates. Related problems are studied in the theory of voting and social choice since the 18 century, however, with no unambiguous solution. The general theoretical conclusion is that no voting rule is perfect. It is proved that every voting rule has its limits, in other words, its 'good' performance is restricted to certain situations (Mueller 1989).

The imperfectness of intermediation of voters' will is to a great extent caused by the irrationality of voters themselves. They are often influenced by *a priori* judgements or pay attention to the image of candidates rather than to real merits. At the business level, some corporations try to overcome subjectivity and irrelevances in evaluation of candidates in recruitment procedures by considering exclusively job-related matters and anonymous questionnaires with no names, photos, or any personal information; see Krause et al. (2010, pp. 8–21) for an international survey. This practice is becoming more widespread, and Germany has started to shape it into juridical guide-lines (Antidiskriminierungsstelle des Bundes 2010).

The given paper discusses these two shortcomings of representative democracy, referring to the German Bundestag elections held on September 27, 2009 (Bundeswahlleiter 2009) with the results displayed in Table 1. Since no party got an absolute majority, the question emerges if the election winner CDU/CSU and the currently governing coalition CDU/CSU/FDP are as desired by the population as prompted by the election outcomes.

Table 1: Results of German parliamentary elections 2009

	CDU/CSU	SPD	FDP	die Linke	Grünen	22 minor parties
Percentage of votes	33.8	23.0	14.6	11.9	10.7	6.0

SPD is the Social Democratic Party

CDU/CSU is the Christian Democratic Union together with Bavaria’s Christian Social Union (conservators)

FDP is the Free Democratic Party (neoliberals) close to employer organizations

die Linke (Left-Party) is a fusion of the PDS (Party of Democratic Socialism—former East German communists) with the WASG (Voting Alternative for Employment and Social Justice—the separated left wing of the SPD)

Grünen (Greens) is the party of ecologists in a broad sense with a social-democratic background

Secondly, the rationality of voters is also put in question, that is, their voting behavior is asked to be really optimal with regard to their own political profiles.

The analysis is performed from the standpoint of direct democracy. The issues declared in the party manifestos are compared with the results of public opinion polls on the same issues. Then the parties and coalitions are evaluated with two indicators of representativeness — *popularity* and *universality*, following (Tangian 2008 and 2010). The indicators are derived from the size of groups resulting from *crosscutting cleavages* (Pitkin 1967, Miller 1964, Wright 1978, Miller 1983, and Brams et al. 1998). Both indicators suggest a kind of correlation measure for estimating the proximity between party positions and voters’ opinions (Achen 1977, 1978). The crosscutting cleavages are determined by a number of dichotomous questions (with Yes/No answers), each dividing the society into two groups, protagonists and antagonists, with positive and negative opinions, respectively. The parties, answering these questions, represent some Yes-groups and some No-groups. The popularity of a party is measured by the size of the group represented, averaged on all the questions selected. The universality of a party is the frequency of representing a majority. One can say that the popularity reflects the spatial aspect of representativeness, and the universality reflects its temporal aspect.

The required information on the party positions at the time of elections 2009 is available from the *Wahl-O-Mat* (Bundeszentrale für politische Bildung 2010). There, the party positions are specified in a tabular form as Yes/No answers to 38 topical questions (Introduce nation-wide minimal wage? Yes/No; Prolong the operation time of nuclear power plants? Yes/No, etc.). Besides, the answers for the DGB (German Confederation of Trade Unions) are provided by the editor of the DGB periodical *Einblick* Anne Graef.

Recall that the Wahl-O-Mat (‘Electomat’) is the German version of the Dutch Internet portal *StemWijzer* (‘VoteMatch’) of the Institute for Public and Politics (2010). Both web-sites help *individuals* to locate themselves at the political landscape by testing the goodness of fit of a potential voter to party positions. Before elections, a special commission formulates a number of questions and addresses them to the parties for reference answers. The visitor of the web-site also provides answers to these questions, eventually with weights, and the computer program, having compared them with the party answers, finds the best-matching party, the second-best matching party, etc.; for details see Bun-

deszentrale für politische Bildung (2010). To avoid political speculations, the individual answers are saved neither in anonymized form, nor as cumulated statistics. Therefore, to test the goodness of fit of parties to the whole of electorate, which is our goal, we use data from different public polls listed in Table 10 in the annex.

In our study, the questions on policy issues are considered either unweighted, or weighted by two experts. The party indices of popularity and universality turn out to be quite similar for the three weighting methods. The explanation is that the party answers are determined by the party “ideology” and are therefore highly correlated, making the overall evaluation little sensitive to question weights. The party indices of popularity and universality show that the winner of the Bundestag elections 2009 — the conservative party CDU/CSU with 33.8% votes — has a quite low representative capacity (fourth among the five leading parties), whereas the most representative is the left party *Linke* which received only 11.9% votes. Its low rating in elections can be explained by a bad image of the former GDR communist party, especially in the West Germany. It is noteworthy that the DGB is also top evaluated as a good representative of majority opinions.

Besides, a kind of coalition formation analysis is performed; for theoretical references see van Deemen (1997) and de Vries (1999). All coalitions with two and three parties are evaluated. The most representative one would consist of *Linke* and *Grünen*, who received together 22.6% votes and therefore could not make a government. It is noteworthy that the currently governing coalition CDU/CSU/FDP is constituted by the two least representative parties among the five. The coalition itself is the least popular and the least universal from all imaginable coalitions.

Thus, the analysis of Bundestag elections 2009 shows that the voters are little consistent with their own political profiles, disregard party manifestos, and are likely driven by political traditions, even if outdated, or by personal images of politicians. A possible explanation is that the spectrum of the German political landscape has significantly shifted to the right, whereas voters still believe that the parties represent the same values as a few decades ago. The insufficient attention to party profiles results in a discrepancy between electorate and elected government. In case of Germany it manifests itself in the unprecedented violent response of population to some rather usual events like starting the construction of a new main station in Stuttgart (Protest gegen Stuttgart 21, 2010), or transporting atomic waste (Castor-Transport geht auf schwierigste Etappe 2010).

Taking into account the results of the study, some modifications to the election procedure are proposed to bridge approaches of representative and direct democracy. It is suggested to introduce the third vote in the form of ‘sample referenda’ with voters’ Yes/No opinions on several important issues from party manifestos. It meets the existing logic of the German two-vote system: the first vote for a person, the second vote for a party, and the third vote for party profiles, so that the considerations are getting to be more conceptual and less personified. An example of application is developed with the data of the German parliamentary elections 2009.

In Section 2, “Indicators of popularity and universality”, initial data (over 30 questions with weights and party answers), basic assumptions, and indicators of popularity and universality of parties are introduced.

In Section 3, “Evaluation of parties”, the indicators of popularity and universality of parties and DGB are calculated.

In Section 4, “Evaluation of coalitions”, the indicators of popularity and universality are extended to coalitions with two and three parties.

In Section 5, “Elections with elements of direct democracy”, a method of elections based on evaluation of candidate profiles with the indices or representativeness is proposed and, for illustration, applied to the data of the German parliamentary elections 2009.

In Section 6, “Conclusions”, the main statements of the paper are recapitulated.

In Section 7, “Annex 1: Computational issues”, the mathematical model is rigorously described and computation formulas are derived.

In Section 8, “Annex 2: Polls of public opinion”, the Internet links to the data sources are provided.

2 Indicators of popularity and universality

Table 2 displays the data for the study. The first section contains Yes/No answers of five leading German parties and of DGB to the 38 Wahl-O-Mat questions grouped into thematic topics. The answers for the parties are given by the parties themselves and are available from the Wahl-O-Mat (Bundeszentrale für politische Bildung 2010). A few missed answers for the questions marked with * are made up from party public statements, voting in the parliament, etc. by Matthias Hölzlein (2010). The answers for the DGB are given by the editor of the DGB periodical *Eimblick* Anne Graef.

The second section of Table 2 contains the question weights in the range 1–5 suggested by two experts from the Hans-Böckler-Stiftung (political foundation of the DGB) — by the director of the Institute for Economic and Social Research Claus Schäfer and by the director of the archive of collective agreements Reinhard Bispink. The last section contains the results of public opinion polls; for the sources see Table 10 in Annex 2. The public polls cover 32 of 38 questions, and only these questions are used in further analysis.

Table 2 is visualized by Figure 2. To explain the figure, consider the top question: ‘2. Introduce nation-wide minimal wage’. Each party is shown by a rectangle with the ‘official’ party color, which length is proportional to the number of the party seats in the Bundestag. The ‘No/Yes’ party opinion on the question is reflected by the location of the rectangle to the left side or to the right side from the central vertical axis, respectively. A Bundestag majority is attained if the cumulative length of party rectangles surpasses the 50%-threshold (marked with dotted lines). The results of the relevant public survey are shown by the blue bar with the length normalized to 100% (abstaining respondents are ignored). Its bias from the center indicates at the prevailing social opinion. The small red rectangle of the DGB has no quantitative meaning but only indicates the ‘Yes/No’ position.

For every question, a given party represents either a majority, or a minority of the population (identified with the fraction in the opinion polls). For instance, the CDU/CSU (black rectangle) with the ‘No’ answer to the top question ‘2. Introduce nation-wide minimal wage’ represents the opinion of 43% of the population against 52%; see Table 2 for exact figures. After normalization, we obtain that its *representativeness* for question 2 is

$$r_{\text{CDU/CSU},2} = \frac{43}{43 + 52} \cdot 100\% \approx 45\% .$$

Similarly, with the ‘No’ answer to the next question ‘17. Relax protection against dismissals’, the CDU/CSU expresses the opinion of 82% of the population against 17%.

The Wahl-O-Mat question number and the question (shortly formulated)	Party positions and votes received, in %						Question weights 1–5		Survey results, in %	
	CDU/CSU	SPD	FDP	Linke	Gruenen	DGB	1st expert	2nd expert	Protagonists	Antagonists
	33.8	23.0	14.6	11.9	10.7	0.0	Schaefer	Bispink		
Labour market										
2. Introduce nation-wide minimal wage	No	Yes	No	Yes	Yes	Yes	5	5	52	43
17. Relax protection against dismissals	No	No	Yes	No	No	No	5	5	17	82
Economy and taxes										
24. Exclusive governmental ownership of railways	No	Yes	No	Yes	Yes	Yes	5	3	70	28
10. Equity holding by government in private banks has to be temporary	Yes	Yes	Yes	No	Yes	Yes	3	3	28	67
5. No state control over top-management salaries*	Yes	Yes	Yes	No	No	No	4	4	30	67
13. Decrease corporate taxes	Yes	No	Yes	No	No	No	5	3	23	59
28. Reintroduce a wealth tax*	No	Yes	No	Yes	Yes	Yes	5	5	35	65
Environment										
1. Prolong the operation time of nuclear power plants	Yes	No	Yes	No	No	No	3	4	39	55
19. Introduce a general speed limit on Autobahnen (German motorways)	No	Yes	No	Yes	Yes	No	1	1	34	41
9. Unexceptionally ban experiments on animals	No	No	No	Yes	Yes	No	1	1	19	80
26. Authorize production of genetically modified food*	Yes	Yes	Yes	No	No	No	3	3	33	67
22. More subsidies for eco-farming	??	Yes	No	Yes	Yes	Yes	4	2	No data	No data

Table 2: Sheet A. Data for the model

* Adjustments to non-specified party positions based on public statements, parliament voting, etc.

The Wahl-O-Mat question number and the question (shortly formulated)	Party positions and votes received, in %						Question weights 1–5		Survey results, in %	
	CDU/CSU	SPD	FDP	Linke	Gruenen	DGB	1st expert	2nd expert	Protagonists	Antagonists
	33.8	23.0	14.6	11.9	10.7	0.0	Schaefer	Bispink		
Social policy										
36. Increase significantly unemployment benefits (Hartz IV)*	No	No	No	Yes	Yes	Yes	5	5	48	36
32. If wages decrease, pensions can be reduced*	No	No	Yes	No	Yes	No	5	4	28	68
31. No Praxisgebuehr (quarterly fee for medical visits)	No	No	Yes	Yes	Yes	Yes	5	3	71	26
15. Compensation to parents who use no public daycares	Yes	No	No	No	No	No	4	2	65	33
Education										
29. Leave the education policy under the authority of the states	Yes	No	Yes	No	No	Yes	5	3	9	81
34. Leave 3 types of schools with different access to further education	Yes	No	Yes	No	No	Yes	5	2	63	31
16. Guarantee an apprenticeship training position for every adolescent*	No	Yes	No	Yes	Yes	Yes	3	4	81	19
7. The first university degree should be free of tuition fees*	No	Yes	No	Yes	Yes	Yes	5	4	53	47
8. Obligatory language test for all children of preschool age	Yes	Yes	Yes	??	Yes	Yes	3	2	No data	No data
20. BAFOeG (aid to students and trainees) regardless of parent income	No	No	Yes	Yes	Yes	Yes	5	3	No data	No data
Gender										
12. There should be a quota for women in leading positions	No	Yes	No	Yes	Yes	Yes	3	2	34	64
27. Full adoption rights for homosexual couples	No	Yes	Yes	Yes	Yes	Yes	3	1	51	49

Table 2: Sheet B. Data for the model

* Adjustments to non-specified party positions based on public statements, parliament voting, etc.

The Wahl-O-Mat question number and the question (shortly formulated)	Party positions and votes received, in %						Question weights 1–5		Survey results, in %	
	CDU/CSU	SPD	FDP	Linke	Gruenen	DGB	1st expert	2nd expert	Protagonists	Antagonists
	33.8	23.0	14.6	11.9	10.7	0.0	Schaefer	Bispink		
Domestic policy										
6. Prohibit secret online surveillance of private computers	No	No	Yes	Yes	Yes	Yes	3	2	39	57
25. Retain the compulsory military service	Yes	No	No	No	No	Yes	3	2	41	53
37. Allow domestic use of German military forces against terrorism	Yes	No	No	No	No	No	3	3	69	28
14. Introduce referenda at the federal level	No	Yes	Yes	Yes	Yes	Yes	3	3	68	26
23. German politics should follow Christian values*	Yes	No	No	No	No	No	1	1	73	20
38. The German democracy is the best form of government	Yes	Yes	Yes	Yes	Yes	Yes	3	2	77	11
35. Municipal voting rights for foreign permanent residents	No	Yes	Yes	Yes	Yes	Yes	3	3	44	42
30. Less restriction on asylum policy	No	No	??	Yes	Yes	Yes	3	2	No data	No data
Foreign policy										
3. Immediate withdrawal of German troops from Afghanistan	No	No	No	Yes	No	Yes	5	2	57	37
11. No trade relations with governments who violate human rights	??	No	??	No	??	No	3	2	No data	No data
33. General export prohibition of military materials	No	No	No	Yes	??	No	3	3	No data	No data
European policy										
4. Germany should leave the European Union	No	No	No	No	No	No	1	4	12	86
18. Turkey should be a full member of the European Union	No	Yes	No	Yes	Yes	Yes	3	3	37	58

Table 2: Sheet C: Data for the model

* Adjustments to non-specified party positions based on public statements, parliament voting, etc.

After normalization we obtain its representativeness for question 17

$$r_{\text{CDU/CSU},17} = \frac{82}{82 + 17} \cdot 100\% \approx 83\% ,$$

and so on. Taking the average representativeness of the CDU/CSU over the 32 questions with known results of public polls, we obtain the index of *popularity* of the party

$$P_{\text{CDU/CSU}} = 52\% .$$

Generally, a higher popularity means that a larger fraction of the electorate is represented.

The frequency of representing a majority ($\geq 50\%$) is defined to be the *universality* of the party. As one can see, the CDU/CSU represents a majority on 15 questions from 32, that is, with the frequency

$$U_{\text{CDU/CSU}} = \frac{15}{32} \cdot 100\% \approx 47\% .$$

A higher universality means that a majority is represented more frequently. For instance the *Linke* represents a majority on 22 of 32 questions, resulting in 69%-universality.

The indices of popularity and universality are also computed with weight coefficients of the questions which reflect their relative importance. Then the popularity is defined to be the weighted average representativeness, and the universality is defined to be the weighted frequency of representing a majority; for details see Section 7.

3 Evaluation of parties

The popularity and universality indices of parties, DGB, and Bundestag (determined by a Bundestag majority) are shown in Table 3 and Figure 3 in three versions: for equally important questions (unweighted), and weighted by two experts. For reference, the bottom row of Table 3 shows the absolute maximum of the indicators which could be attained if majority opinions were represented on all the questions. Besides, Table 3 gives ranks of the indices in every column. The fractional rank 2.5 in the first universality column means that Linke and DGB share the second and third places.

Which conclusions do follow from the indices computed?

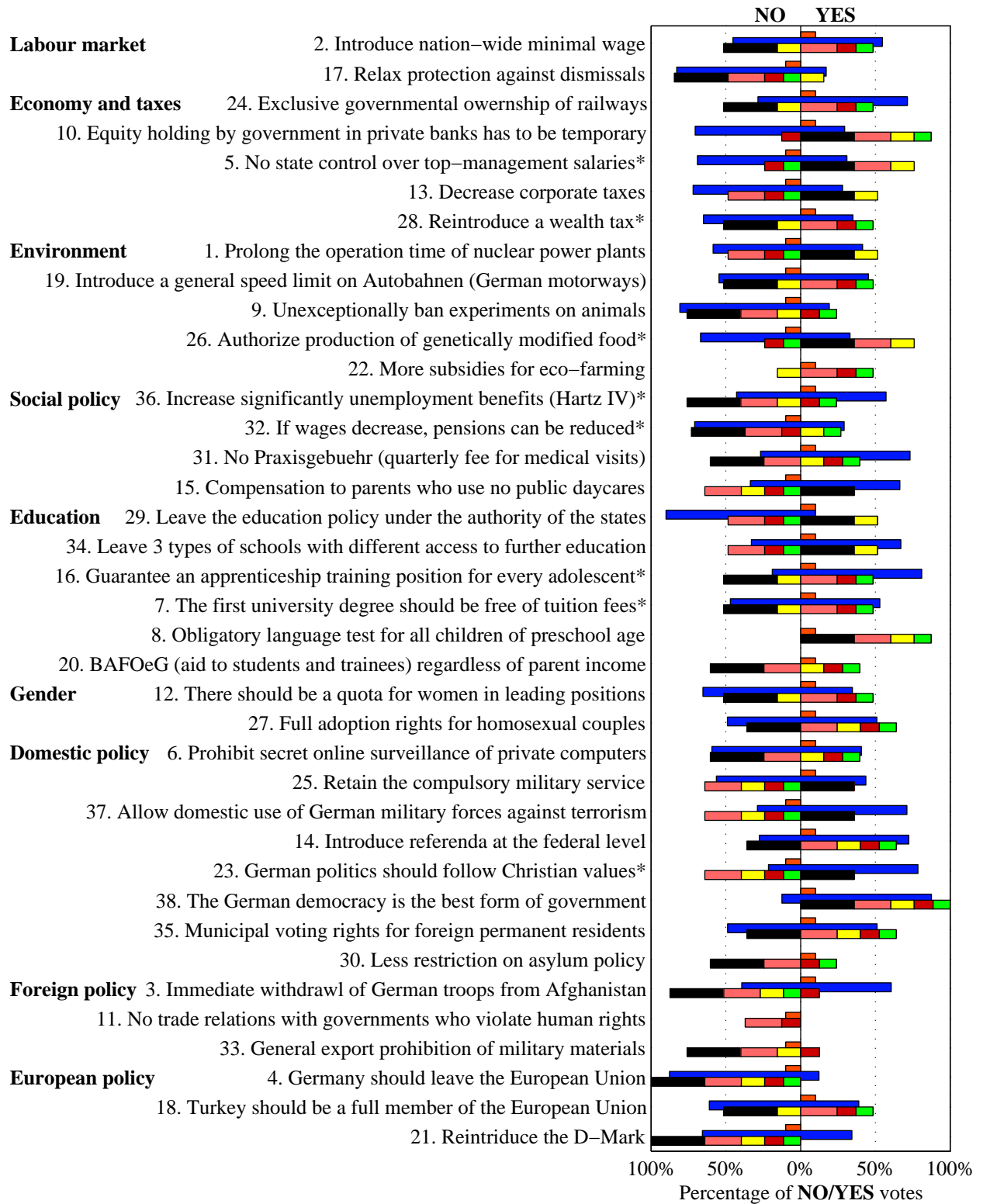
- **Inconsistency of election results with public opinion**

The party indices of popularity and universality show that the winner of the Bundestag elections 2009 — the conservative party CDU/CSU with 33.8% votes — has a quite low representative capacity (fourth among the five leading parties). The second actually governing party — the neo-liberal FDP — is bottom-ranked in all the evaluations. The latter is explained by the fact that the business-friendly FDP represents rather employers, who constitute a minority of the population.

The most representative is the left party *Linke* which received only 11.9% votes. In spite of a high representative capacity, the *Linke* got almost three times fewer votes than the CDU/CSU. It can be explained by the traditional orientation of German voters towards two major parties — CDU/CSU and SPD, by a bad image of the former GDR communist party, especially in the West Germany, and not least by a little attention of voters to party manifestos.

Figure 2: What do the voters give their votes for?

Balance of opinions in the society and its representation by party seats in the Bundestag and by the DGB
 CDU/CSU ■, SPD ■, FDP ■, Linke ■, Gruenen ■, DGB ■, Opinions in public surveys ■



* Adjustments to non-specified party positions based on public statements, parliament voting, etc.

Table 3: Indices of parties, DGB, and Bundestag with their ranks

	Votes	Popularity			Universality		
	%	Unweighted	1st Expert	2nd Expert	Unweighted	1st Expert	2nd Expert
CDU/CSU	33.8	52 / 6	49 / 6	50 / 6	47 / 7	39 / 7	42 / 7
SPD	23.0	54 / 5	53 / 5	55 / 5	56 / 5	55 / 5	59 / 5
FDP	14.6	47 / 8	44 / 8	45 / 8	44 / 8	36 / 8	37 / 8
Linke	11.9	57 / 2	59 / 2	61 / 2	69 / 2.5	74 / 2	77 / 2
Gruenen	10.7	54 / 4	55 / 4	57 / 4	59 / 4	63 / 4	67 / 4
DGB		56 / 3	56 / 3	58 / 3	69 / 2.5	71 / 3	73 / 3
Bundestag		50 / 7	47 / 7	49 / 7	50 / 6	43 / 6	45 / 6
Abs. limit		68 / 1	67 / 1	68 / 1	100 / 1	100 / 1	100 / 1

- **Secondary role of weighting**

As seen from Table 3, the three weightings of the questions do not affect the indicators’ order. The ranks of the party indices are the same for all the parties and DGB. A minor difference in universality ranks is caused by the fact that both the *Linke* and DGB have the same index of unweighted universality 67% and both get the same rank 2.5.

The similarity in index orders can be explained as follows. Answers of a party are backed up by the party “ideology” which determines a high intra-question correlations. Therefore, ‘erroneous’-weighting and even omitting some questions play a rather negligible role, because other questions carry superfluous information on the party position. *Henceforth, only unweighted indicators will be considered.*

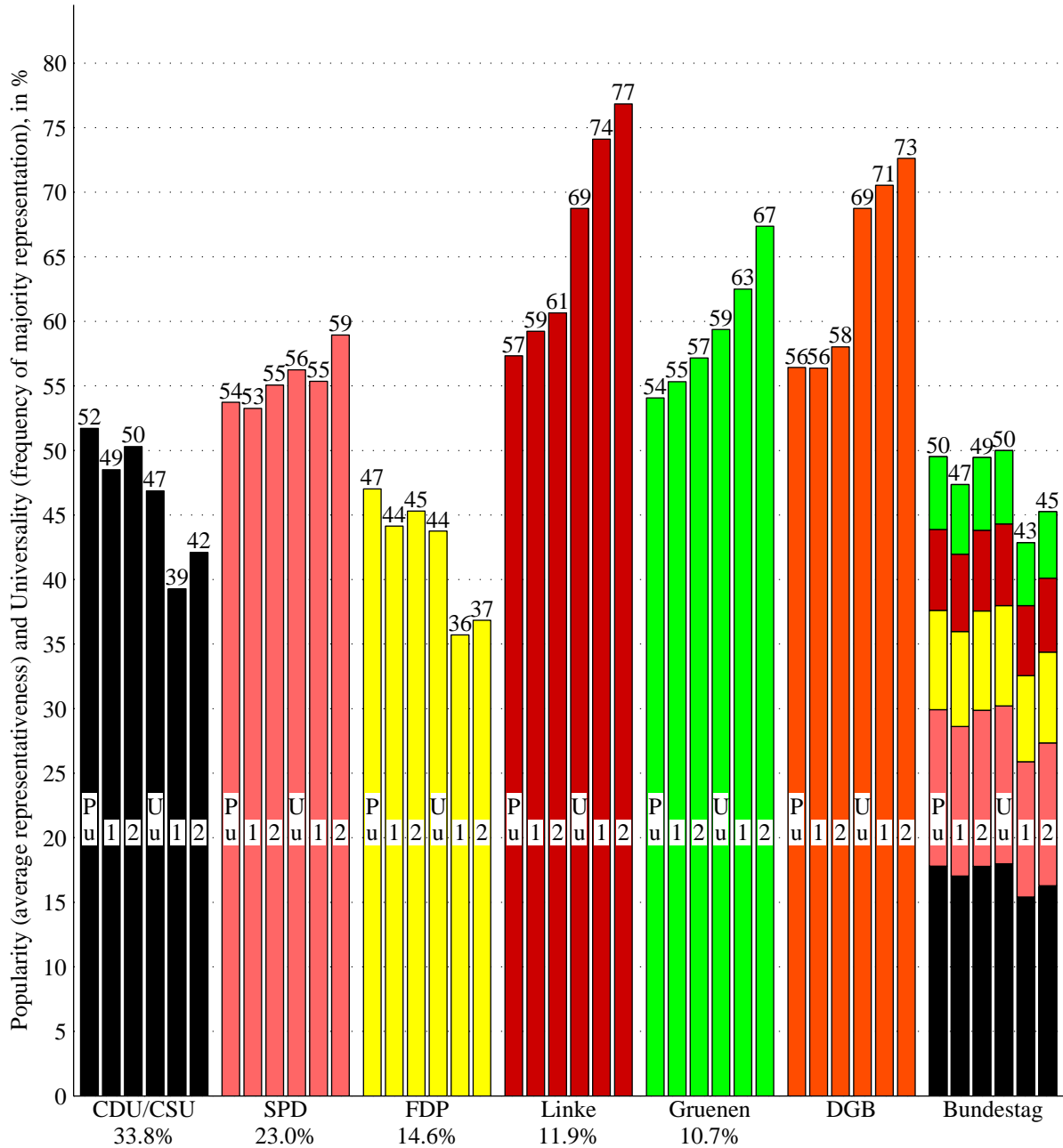
Note that the weighted indices are lower for CDU/CSU and FDP, higher for the *Linke* and *Grünen*, and rather constant for SPD and DGB. It says that the experts give less weights to the issues where the conservative or business-friendly party is highly representative, and give more weights to the issues where the left party and the Greens express the opinion of a majority. In case of politically centrally-located SPD and DGB, the expert weighting does not change much the total balance between more and less popular opinions.

- **Evaluation of representativeness with no dedicated surveys**

The DGB position on party manifestos allows us to evaluate its popularity and universality, although DGB does not participate in elections. The high indices of DGB mean that trade unions are top-representative in Germany, expressing majority opinions on most issues.

Thus, the representativeness of any political body can be evaluated without elections, just by comparing its position with the results of public opinion polls.

Figure 3: Indices of parties, DGB, and Bundestag with their ranks; P – popularity, U – universality, (u) for unweighted questions, (1) weighted by the 1st expert, and (2) weighted by the second expert



4 Evaluation of coalitions

Table 4 displays the parties and all imaginable coalitions with up to three parties together with their indices of popularity and universality for unweighted questions (as explained previously, weighting plays a secondary role). The first column contains the names of parties which constitute the coalition. The second column shows the coalition weight in % of parliament seats¹. For example, the first two-party coalition CDU/CSU/SPD has the percentage of parliament seats

$$\text{Parliamentary weight of CDU/CSU/SPD} = \frac{33.8 + 23.0}{94.0} \cdot 100\% = 60.4\% .$$

The third column shows the degree of *unanimity* of the coalition, expressed in % of questions on which all the coalition members agree, also with ranking. Obviously, single parties are 100%-unanimous and get the 1st rank. The most unanimous coalitions are: Linke/Grünen (rank 2, 90.6% = 29/32 questions), SPD/Grünen (rank 3, 78.1% = 25/32 questions), SPD/Linke (rank 4, 75% = 24/32 questions), and SPD/Linke/Grünen (rank 5, 71.9% = 23/32 questions).

One can suppose that if a coalition is not unanimous on a certain question then the probabilities of its Yes/No answers are proportional to the protagonist-to-antagonist ratio within the coalition (ratio of party weights expressed in parliament seats). However, as evidenced by politicians, the reality is even more uncertain. To deal with the uncertainty, introduce the parameter p — *proportionality of party impact to party weights*.

For example, let the protagonist-to-antagonist ratio within a coalition be 3 : 1. The $p = 1$ means the full probabilistic impact of party weights, that is, the larger party determines the coalition opinions with the probability $\frac{3}{3+1} = \frac{3}{4}$, and the smaller party with probability $\frac{1}{4}$. The $p = 0$ means no probabilistic impact of party weights, that is, each of alternative opinions is accepted with equal chances $\frac{1}{2}$. The $p = \frac{1}{3}$ means that the probabilistic impact of party weights is a mix of the two extreme cases, so that the Yes/No answers are adopted by the coalition with the following probabilities

$$\begin{aligned} \text{Probability of 'Yes'} &= \underbrace{\frac{3}{4}}_{\substack{\text{full} \\ \text{weight} \\ \text{impact}}} \cdot \underbrace{\frac{1}{3}}_p + \underbrace{\frac{1}{2}}_{\substack{\text{no} \\ \text{weight} \\ \text{impact}}} \cdot \underbrace{\left(1 - \frac{1}{3}\right)}_{\substack{1-p \\ \text{mix} \\ \text{factor}}} = \frac{7}{12} \\ \text{Probability of 'No'} &= \frac{1}{4} \cdot \frac{1}{3} + \frac{1}{2} \cdot \left(1 - \frac{1}{3}\right) = \frac{5}{12} . \end{aligned}$$

Throughout the paper, a medium uncertainty is assumed, and $p = \frac{1}{2}$ is applied to all coalitions considered.

Under this assumption, both indicators of popularity and universality turn out to be random variables. The coalition's popularity and universality are understood, respectively, as the expected size of the voter group represented, and as the expected frequency of representing a majority. These indices are given in the corresponding columns 'Expectation' of Table 4. The indices are also characterized by their standard deviations,

¹The figures result from the reduction of votes for parties to the total votes for the parties in the parliament, here 94% (small adjustments of the number of parliament seats prescribed by the German constitution are not taken into account). Therefore, the coalition CDU/CSU/FDP with 48.4% votes has 51.5% parliament seats, constituting the parliament majority.

Table 4: Indices of parties and coalitions with up to three parties for unweighted questions; the coefficient of impact of member weights on the coalition decisions $p = 0.50$

Nr. Members of the coalition	Parliament seats	Unanimity	Popularity		Universality	
			Expectation	Standard deviation	Expectation	Standard deviation
	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank
1 CDU/CSU	36.0 / 17	100.0 / 1	51.7 / 18	$\pm 0.0 / 1$	46.9 / 23	$\pm 0.0 / 1$
2 SPD	24.5 / 21	100.0 / 1	53.7 / 9	$\pm 0.0 / 1$	56.3 / 7	$\pm 0.0 / 1$
3 FDP	15.5 / 23	100.0 / 1	47.0 / 25	$\pm 0.0 / 1$	43.8 / 25	$\pm 0.0 / 1$
4 Linke	12.7 / 24	100.0 / 1	57.3 / 1	$\pm 0.0 / 1$	68.8 / 1	$\pm 0.0 / 1$
5 Gruenen	11.4 / 25	100.0 / 1	54.1 / 5	$\pm 0.0 / 1$	59.4 / 5	$\pm 0.0 / 1$
6 CDU/CSU/SPD	60.4 / 6	40.6 / 9	52.6 / 12	$\pm 2.5 / 7$	51.1 / 18	$\pm 6.8 / 9$
7 CDU/CSU/FDP	51.5 / 9	65.6 / 6	49.8 / 24	$\pm 2.0 / 6$	45.6 / 24	$\pm 5.1 / 6$
8 CDU/CSU/Linke	48.6 / 11	15.6 / 14	53.8 / 8	$\pm 2.9 / 16$	55.2 / 12	$\pm 7.9 / 18$
9 CDU/CSU/Gruenen	47.3 / 13	18.8 / 13	52.6 / 13	$\pm 2.9 / 15$	51.5 / 17	$\pm 7.7 / 15$
10 SPD/FDP	40.0 / 14	50.0 / 7	50.8 / 22	$\pm 2.5 / 8$	50.7 / 20	$\pm 6.2 / 7$
11 SPD/Linke	37.1 / 16	75.0 / 4	55.2 / 3	$\pm 1.6 / 4$	61.5 / 3	$\pm 4.4 / 4$
12 SPD/Gruenen	35.9 / 18	78.1 / 3	53.9 / 7	$\pm 1.6 / 3$	57.5 / 6	$\pm 4.1 / 3$
13 FDP/Linke	28.2 / 19	37.5 / 10	51.9 / 17	$\pm 2.8 / 12$	55.6 / 11	$\pm 7.0 / 12$
14 FDP/Gruenen	26.9 / 20	46.9 / 8	50.3 / 23	$\pm 2.6 / 9$	51.0 / 19	$\pm 6.4 / 8$
15 Linke/Gruenen	24.0 / 22	90.6 / 2	55.7 / 2	$\pm 1.0 / 2$	64.2 / 2	$\pm 2.7 / 2$
16 CDU/CSU/SPD/FDP	76.0 / 1	28.1 / 12	51.4 / 19	$\pm 2.9 / 13$	48.8 / 22	$\pm 7.4 / 14$
17 CDU/CSU/SPD/Linke	73.1 / 2	15.6 / 14	53.9 / 6	$\pm 3.0 / 18$	55.8 / 10	$\pm 8.0 / 19$
18 CDU/CSU/SPD/Gruenen	71.8 / 3	18.8 / 13	52.8 / 11	$\pm 3.0 / 17$	52.6 / 15	$\pm 7.8 / 16$
19 CDU/CSU/FDP/Linke	64.1 / 4	9.4 / 16	52.3 / 15	$\pm 3.2 / 21$	52.6 / 16	$\pm 8.1 / 20$
20 CDU/CSU/FDP/Gruenen	62.9 / 5	15.6 / 14	51.4 / 20	$\pm 3.1 / 20$	50.0 / 21	$\pm 7.9 / 17$
21 CDU/CSU/Linke/Gruenen	60.0 / 7	12.5 / 15	53.6 / 10	$\pm 3.1 / 19$	55.1 / 13	$\pm 8.2 / 21$
22 SPD/FDP/Linke	52.7 / 8	31.3 / 11	52.2 / 16	$\pm 2.9 / 14$	55.9 / 9	$\pm 7.2 / 13$
23 SPD/FDP/Gruenen	51.4 / 10	37.5 / 10	51.3 / 21	$\pm 2.8 / 10$	53.1 / 14	$\pm 6.8 / 10$
24 SPD/Linke/Gruenen	48.5 / 12	71.9 / 5	54.8 / 4	$\pm 1.8 / 5$	60.6 / 4	$\pm 4.6 / 5$
25 FDP/Linke/Gruenen	39.6 / 15	37.5 / 10	52.3 / 14	$\pm 2.8 / 11$	56.2 / 8	$\pm 6.9 / 11$

Table 5: Correlations ρ between the indicators and P-values

	For all coalitions			For coalitions with $> 50\%$ seats		
	Popularity	Universality	Unanimity	Popularity	Universality	Unanimity
Popularity	1.00 / 0.00	0.91 / 0.00	0.16 / 0.45	1.00 / 0.00	0.81 / 0.00	-0.71 / 0.02
Universality		1.00 / 0.00	0.26 / 0.20		1.00 / 0.00	-0.62 / 0.06
Unanimity			1.00 / 0.00			1.00 / 0.00

which can be interpreted as the prediction accuracy. As one can see, the highest expected indices of properly coalitions, as well as the best prediction accuracy (all with rank 2, since rank 1 has the party Linke) are inherent in the coalition Linke/Grünen.

The correlation between the three indicators is shown in Table 5. The indices of popularity and universality are highly correlated, which is explained by their formal definitions (2) and (3). Their correlation with the unanimity indices is not that high, meaning that the unanimity of a coalition has little to do with its representative capacity. The location of coalitions in the space Popularity–Universality–Unanimity is depicted in Figures 4 and 5, where coalitions are aligned along the diagonal in the bottom plane, exhibiting the correlation between the indices of popularity and universality.

The main findings are as follows:

- **The size of coalition is not a criterion of representativeness**

Figure 4 illustrates that the *smallest* coalition Linke/Grünen is the most representative with regard to both indicators. Since both parties received together 24% parliament seats, the coalition could not rule, whereas the currently governing coalition CDU/CSU/FDP with 51.5% parliament seats is the least popular and the least universal among *all* coalitions, to say nothing about large coalitions with a majority of parliament seats shown in Figure 5.

Therefore, the coalition size is not an adequate criterion of coalition representativeness.

- **Coalition formation can reduce the parliament representativeness**

The actual practice of coalition formation aimed at attaining a parliamentary majority can decrease the representativeness of the elected parliament. Indeed, popularity and universality indices of the elected parliament are both 50% (see Table 3 and Figure 3), whereas these indices for the currently governing coalition CDU/CSU/FDP are 49.8 and 45.6%, respectively.

This means that the practice of coalition formation can contradict the objectives of representative democracy.

5 Elections with elements of direct democracy

The findings of the paper show that the performance of representative democracy regarded from the viewpoint of direct democracy is far from being perfect. At the same time, the evaluation framework developed in the paper prompts a way to improve the election procedure. In case of Germany, one can imagine the third vote in the form of 'sample referenda' with voters' Yes/No opinions on several important issues from party manifestos. It meets the existing logic of the German two-vote system: the first vote for a person, the second vote for a party, and the third vote for party profile, so that the considerations are getting to be more conceptual and less personified.

The aiming is (a) to redirect the voters' attention from candidates as persons to their manifestos as political profiles, and (b) to base the election of candidates on matching their profiles to the majority will. Technically, ballots could contain a number of Yes/No questions asking for the voter position on the issues in the candidate manifestos. As

Figure 4: Indices of parties and coalitions with up to three parties for unweighted questions; the coefficient of impact of member weights on the coalition decisions $p = 0.50$

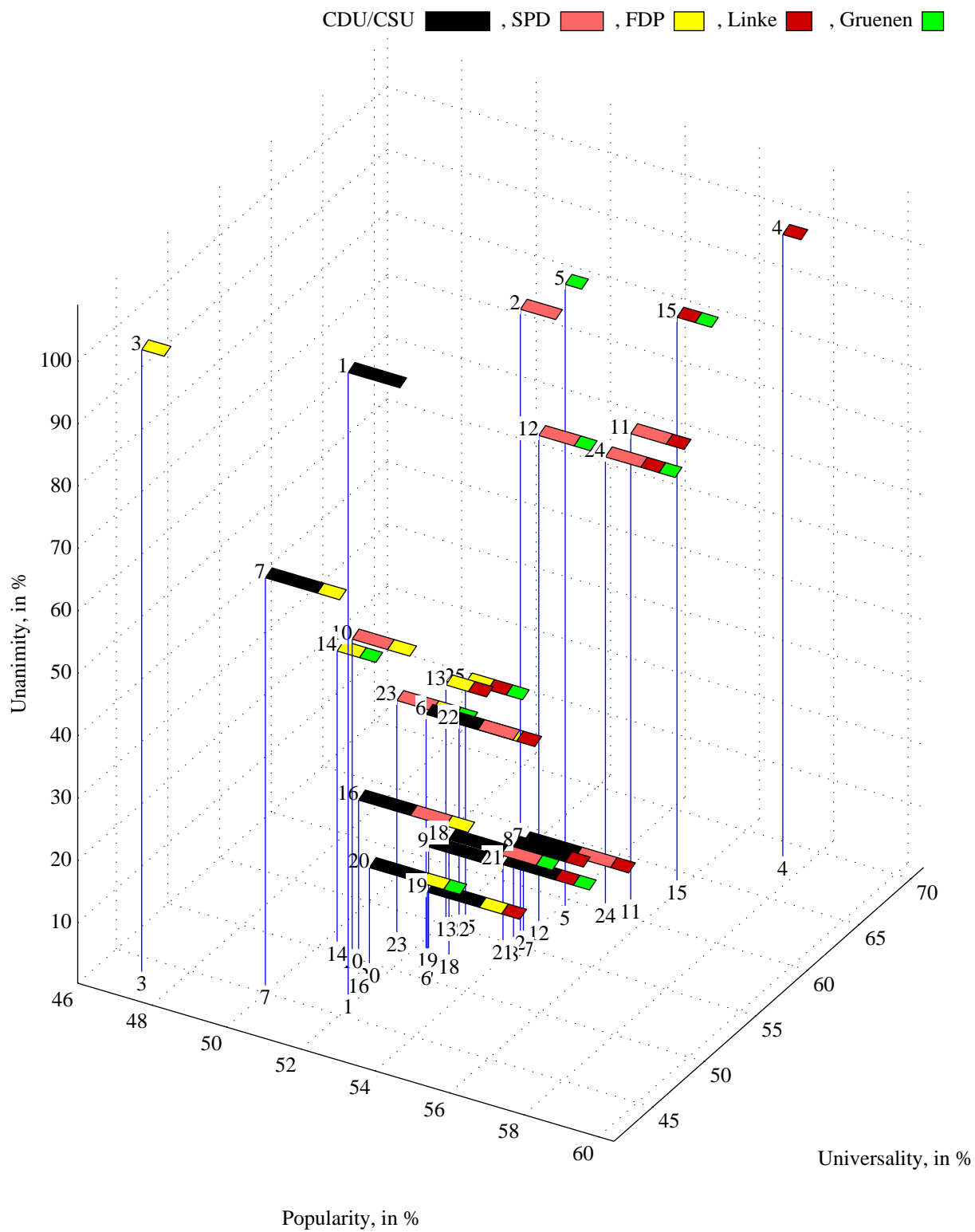
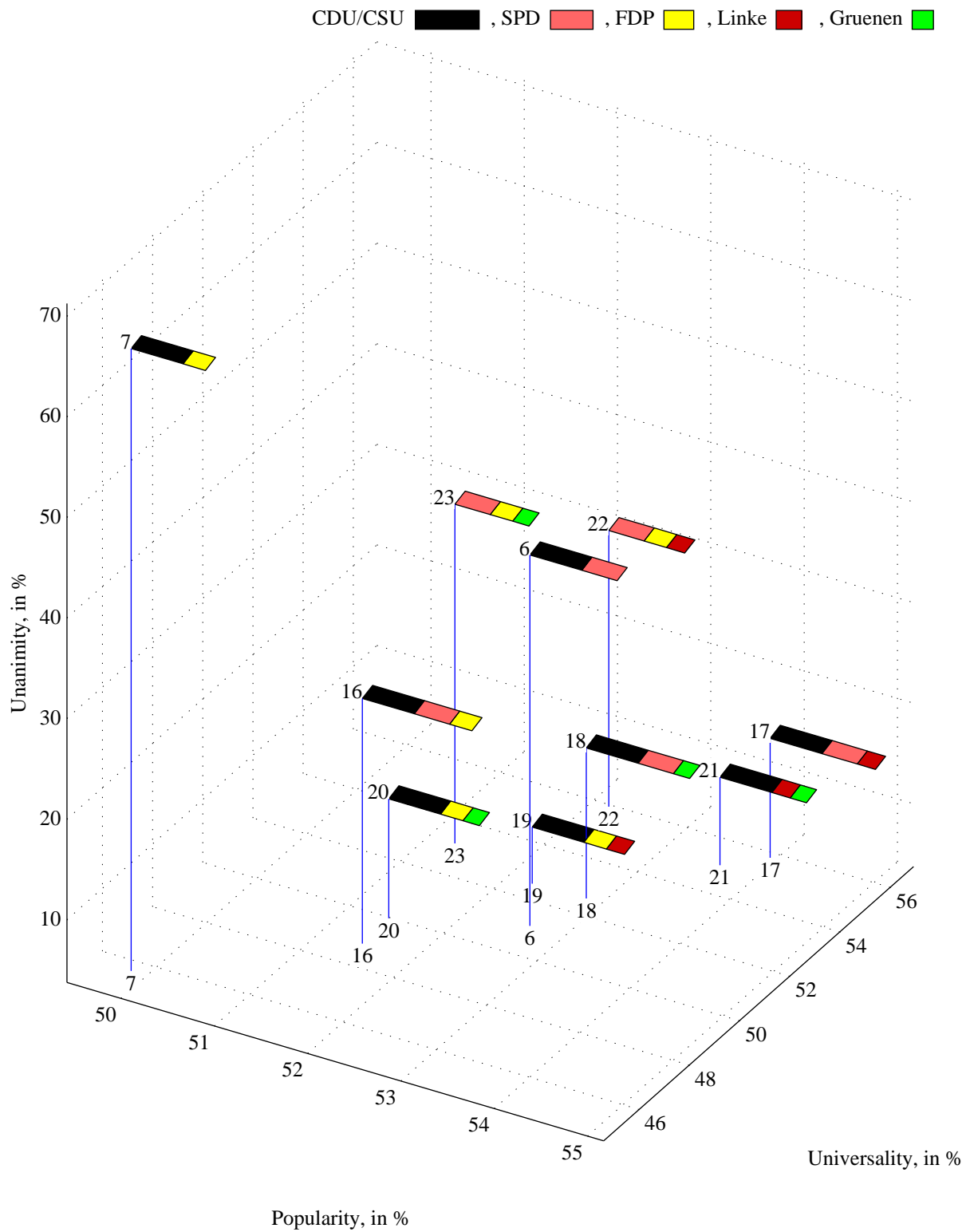


Figure 5: Indices of parties and coalitions with up to three parties having at least half the parliament seats for unweighted questions; the coefficient of impact of member weights on the coalition decisions $p = 0.50$



mentioned earlier, there is a strong correlation between answers to selected questions determined by party ideologies, so that a sample of questions provides a quite reliable basis for specifying political profiles both of voters and candidates. Parties themselves can formulate the important questions and specify their positions.

The first method of election is based on processing each single ballot and finding the best-matching candidate who then receives the given vote. It does not change the election procedure itself, but only enhances the purposeful dedication of votes. Here, the irrational behavior of voters can be overcome. In fact, this method is implemented in the advisory option of the Wahl-O-Mat.

The second method is based on processing the totality of ballots. After the balance of electorate opinions on the issues has been revealed, the candidates should be matched to the profile of the *whole of electorate*. The evaluation of their representative capacity can be done with aggregate indices like popularity or universality used as a reference to distribute parliament seats. This method is equivalent to performing a series of referenda and integrates elements of direct democracy in the election procedure. Thereby no candidate little desired by a majority can be elected, and no cyclic orders can emerge (Mueler 1989), because candidates are indexed, and indices have no cycles.

Let us illustrate the second method with the data from Table 2². First of all select the 'most important' reference indicator. For this purpose, apply principle component analysis to the parties and coalitions in the Popularity–Universality–Unanimity space shown in Figures 4 and 5³. As seen from Tables 6 and 7, the contribution of universality absolutely predominates in the first two (most important) components. Consequently, the universality is regarded as more important than popularity.

Now 'adjust votes' for parties (in fact, new parliamentary weights) proportionally to the party (unweighted) universality indices from Table 3 and Figure 3. For instance,

$$\text{Adjusted votes for CDU/CSU} = \frac{\overbrace{47}^{\text{Universality of CDU/CSU}}}{\underbrace{47 + 56 + 44 + 69 + 59}_{\text{Sum of universality indices of the five leading parties}}} \cdot 100\% = 17\% .$$

Proceeding in the same way, we obtain the column 'Votes' in Table 8. The indices of Popularity and Universality of the parties remain obviously the same, and only the Bundestag indicators are adjusted to the new number of party seats. Repeating the computations from Section 4 for coalitions with 'adjusted votes', we obtain Table 9 and Figure 6.

Now no two parties build a governing coalition, and every three-party coalition includes the 'election winner' — the Linke. The most representative coalition is SPD/Linke/Grünen, that is, contains the three most representative parties from Table 3 and Figure 3.

²It is not possible to illustrate the first method with real figures, because individual opinions on policy issues are available neither from Wahl-O-Mat, nor from public opinion polls.

³Principle component analysis is aimed at reducing the dimensionality of observations with least losses of information (here, from two dimensions of parties and coalitions — popularity and universality — only one has to be retained). For this purpose, the location of the cloud of observations is approximated by an ellipsoid, which first diameter is the vector of the maximal variance, the second diameter is the vector of the second largest variance, etc. (Jackson 1988, Krzanowski 1988, and Seber 1984). The largest diameter is regarded as the main component in the observations, indicating at 'most important' dimensions.

Table 6: Principal components for three indicators

	For all coalitions			For coalitions with > 50% seats		
	First compo- nent	Second compo- nent	Third compo- nent	First compo- nent	Second compo- nent	Third compo- nent
Popularity	0.0107	0.3314	0.9434	-0.0507	0.2230	0.9735
Universality	0.0472	0.9423	-0.3316	-0.1198	0.9663	-0.2276
Unanimity	0.9988	-0.0481	0.0056	0.9915	0.1282	0.0222
Standard deviation w.r.t. new axes	33.0478	5.8496	0.8108	17.3108	2.6199	0.6124

Table 7: Principal components for indicators of popularity and universality only

	For all coalitions		For coalitions with > 50% seats	
	First component	Second component	First component	Second component
Popularity	0.3241	0.9460	0.2987	0.9543
Universality	0.9460	-0.3241	0.9543	-0.2987
Standard deviation w.r.t. new axes	6.0551	0.8302	3.4262	0.6779

Table 8: Indices of parties, DGB, and Bundestag with their ranks if the votes for parties were proportional to their universality

	Votes	Popularity			Universality		
	%	Unweighted	1st Expert	2nd Expert	Unweighted	1st Expert	2nd Expert
CDU/CSU	17.0	52 / 7	49 / 7	50 / 7	47 / 7	39 / 7	42 / 7
SPD	20.5	54 / 6	53 / 6	55 / 6	56 / 5.5	55 / 6	59 / 6
FDP	15.9	47 / 8	44 / 8	45 / 8	44 / 8	36 / 8	37 / 8
Linke	25.0	57 / 2	59 / 2	61 / 2	69 / 2.5	74 / 2	77 / 2
Gruenen	21.6	54 / 5	55 / 4	57 / 4	59 / 4	63 / 4	67 / 4
DGB		56 / 3	56 / 3	58 / 3	69 / 2.5	71 / 3	73 / 3
Bundestag		55 / 4	55 / 5	56 / 5	56 / 5.5	57 / 5	60 / 5
Abs. limit		68 / 1	67 / 1	68 / 1	100 / 1	100 / 1	100 / 1

Table 9: Indices of parties and coalitions with up to three parties if the votes for parties were proportional to their universality for unweighted questions; the coefficient of impact of member weights on the coalition decisions $p = 0.50$

Nr. Members of the coalition	Parliament seats	Unanimity	Popularity		Universality	
			Expec- tation	Stan- dard devia- tion	Expec- tation	Stan- dard devia- tion
	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank	%/Rank
1 CDU/CSU	17.0 / 21	100.0 / 1	51.7 / 18	±0.0 / 1	46.9 / 23	±0.0 / 1
2 SPD	20.5 / 20	100.0 / 1	53.7 / 10	±0.0 / 1	56.3 / 13	±0.0 / 1
3 FDP	15.9 / 22	100.0 / 1	47.0 / 25	±0.0 / 1	43.8 / 25	±0.0 / 1
4 Linke	25.0 / 18	100.0 / 1	57.3 / 1	±0.0 / 1	68.8 / 1	±0.0 / 1
5 Gruenen	21.6 / 19	100.0 / 1	54.1 / 8	±0.0 / 1	59.4 / 5	±0.0 / 1
6 CDU/CSU/SPD	37.5 / 15	40.6 / 9	52.8 / 15	±2.5 / 7	51.8 / 19	±6.8 / 10
7 CDU/CSU/FDP	33.0 / 17	65.6 / 6	49.4 / 24	±2.0 / 6	45.3 / 24	±5.2 / 6
8 CDU/CSU/Linke	42.0 / 12	15.6 / 14	54.8 / 5	±3.0 / 18	58.8 / 6	±8.1 / 20
9 CDU/CSU/Gruenen	38.6 / 14	18.8 / 13	53.0 / 12	±3.0 / 17	53.5 / 17	±8.0 / 16
10 SPD/FDP	36.4 / 16	50.0 / 7	50.6 / 23	±2.5 / 8	50.4 / 21	±6.2 / 7
11 SPD/Linke	45.5 / 10	75.0 / 4	55.6 / 3	±1.7 / 4	62.8 / 3	±4.4 / 4
12 SPD/Gruenen	42.0 / 11	78.1 / 3	53.9 / 9	±1.6 / 3	57.8 / 9	±4.1 / 3
13 FDP/Linke	40.9 / 13	37.5 / 10	52.7 / 16	±2.8 / 12	57.6 / 11	±6.9 / 12
14 FDP/Gruenen	37.5 / 15	46.9 / 8	50.8 / 22	±2.6 / 9	52.2 / 18	±6.4 / 8
15 Linke/Gruenen	46.6 / 9	90.6 / 2	55.8 / 2	±1.0 / 2	64.2 / 2	±2.7 / 2
16 CDU/CSU/SPD/FDP	53.4 / 8	28.1 / 12	51.3 / 21	±2.9 / 14	49.0 / 22	±7.4 / 14
17 CDU/CSU/SPD/Linke	62.5 / 3	15.6 / 14	54.6 / 6	±3.0 / 16	58.3 / 7	±8.0 / 17
18 CDU/CSU/SPD/Gruenen	59.1 / 5	18.8 / 13	53.1 / 11	±3.0 / 15	53.9 / 15	±7.8 / 15
19 CDU/CSU/FDP/Linke	58.0 / 6	9.4 / 16	52.8 / 14	±3.2 / 21	55.1 / 14	±8.3 / 21
20 CDU/CSU/FDP/Gruenen	54.5 / 7	15.6 / 14	51.6 / 19	±3.1 / 20	51.2 / 20	±8.0 / 18
21 CDU/CSU/Linke/Gruenen	63.6 / 2	12.5 / 15	54.3 / 7	±3.0 / 19	58.0 / 8	±8.1 / 19
22 SPD/FDP/Linke	61.4 / 4	31.3 / 11	52.6 / 17	±2.9 / 13	57.2 / 12	±7.2 / 13
23 SPD/FDP/Gruenen	58.0 / 6	37.5 / 10	51.4 / 20	±2.8 / 11	53.6 / 16	±6.8 / 11
24 SPD/Linke/Gruenen	67.0 / 1	71.9 / 5	55.0 / 4	±1.8 / 5	61.4 / 4	±4.6 / 5
25 FDP/Linke/Gruenen	62.5 / 3	37.5 / 10	52.9 / 13	±2.8 / 10	57.7 / 10	±6.8 / 9

Of course, this example is provided just for illustration. A practical implementation should not exclude traditional ways of expressing opinions. Additionally to questionnaires in the ballots, a direct vote for a candidate should be the option. Note that such a voting duality is already inherent in the German parliamentary election system with the first vote for a specific person, and the second vote for a party. In our consideration, the vote for a party is replaced by a vote for an even more impersonal party manifesto.

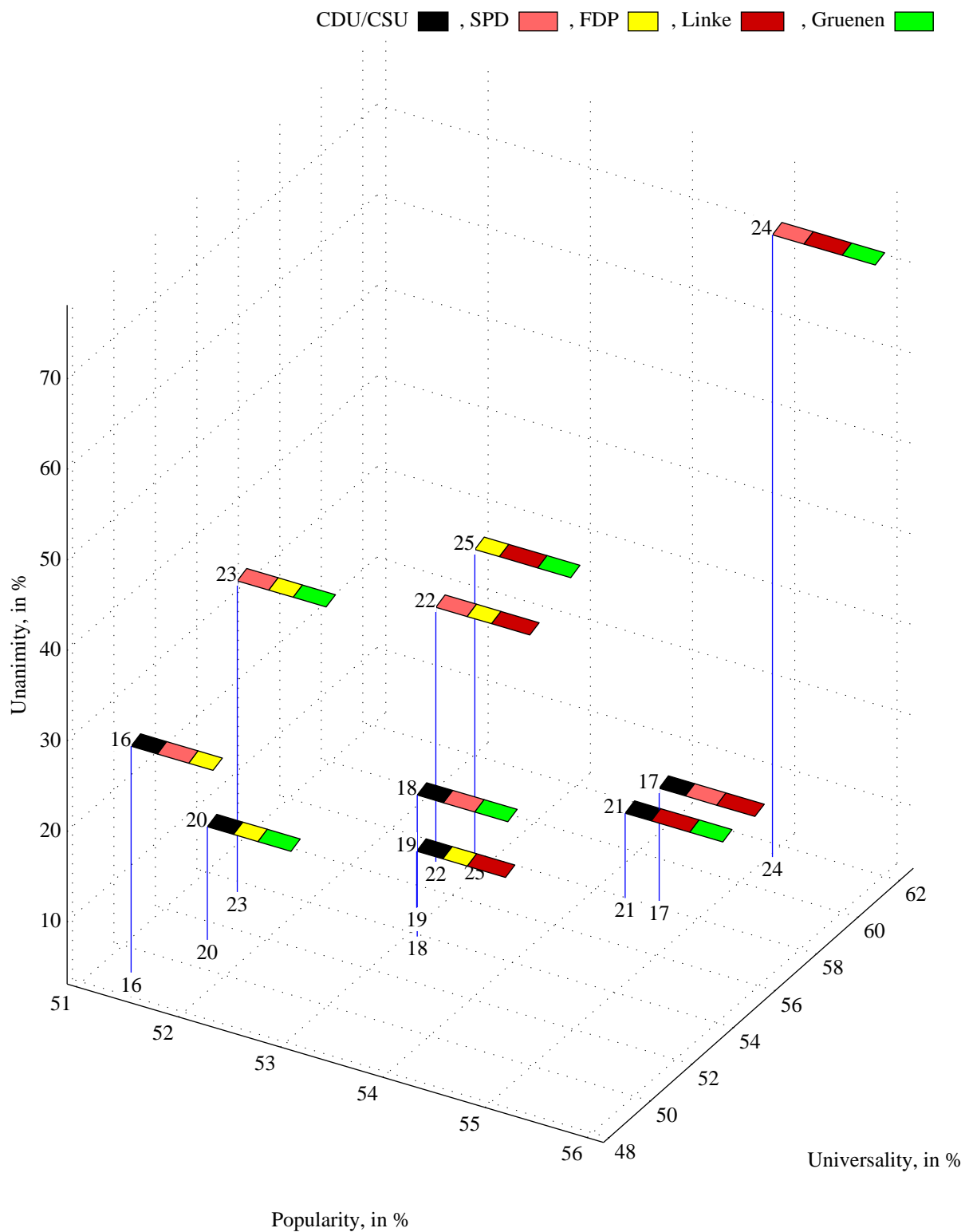
6 Conclusions

Our analysis of the German Bundestag elections 2009 shows that voters are little consistent with their own political profiles, disregard party manifestos, and are likely driven by political traditions, even if outdated, or by personal images of politicians. A possible explanation is that the spectrum of the German political landscape has significantly shifted to the right, whereas voters still believe that the parties represent the same values as a few decades ago. All of these result in election of 'wrong' parties and 'wrong' coalition formation. In case of the German parliamentary elections 2009, the two governing parties are the least representative among the five leading ones, and the governing coalition CDU/CSU/FDP is the least representative among all imaginable coalitions. The insufficient focus on the changing party profiles results in a discrepancy between electorate and elected government. In case of Germany it manifests itself in the increasing social tension and violent protests.

Taking into account the results of the study, some modifications to the election procedure are proposed to bridge approaches of representative and direct democracy. It is suggested to introduce the third vote in the form of 'sample referenda' with voters' Yes/No opinions on several important issues from party manifestos. It meets the existing logic of the German two-vote system: the first vote for a person, the second vote for a party, and the third vote for party profiles, so that the considerations are getting to be more conceptual and less personified.

Taking into account the results of the study, we conclude that voting for candidates or for parties can violate the principle of equal chances, because the parties who are long at power are over-represented, and the social will is not adequately reflected. Following the actual trends in job recruitment procedures with anonymized applications and the focus on job-relevant merits rather than on personal information, the election procedure could be also complemented with methods of revealing the electorate opinion. The voters could be (optionally) asked to answer a number of questions relevant to party manifestos ('sample referenda') to the end of specifying their political profiles. It meets the existing logic of the German two-vote system: the first vote for a person, the second vote for a party, and the third vote for party profiles, so that the considerations are getting to be more conceptual and less personified. Such an approach could integrate elements of most advantageous direct democracy into functionally better manageable representative democracy.

Figure 6: Indices of parties and coalitions up to three parties having at least half the parliament seats if the votes for parties were proportional to their universality; the coefficient of impact of member weights on the coalition decisions $p = 0.50$



7 Annex 1: Computational issues

This section introduces notation and formulas for evaluating parties and their coalitions. Coalition analysis in terms of stable governments has been considered by Rusinowska et al. (2005). Relational algebra (Schmidt and Ströchlein 1993, Brink et al. 1997) is used to compute the stable governments in (Rusinowska et al. 2005 and 2006) and (Berghammer et al. 2007). The vector-matrix formulas adduced below are simpler and have a clear geometric interpretation.

Questions/Agenda By Q denote the agenda with dichotomous questions q , that is, which evoke either positive or negative *opinions* (Yes/No answers) coded by ± 1 . In our applications, the list of $m = 38$ questions is given in the first section of Table 2.

The importance of questions is reflected by weights μ_q which constitute a *probability measure* μ on Q (the reference to “probability” can be misleading; in fact, we just need a normalized additive measure). It assumes

non-negativity

$$\mu_q \geq 0 \quad \text{for every } q \in Q ,$$

additivity

$$\mu_X = \sum_{q \in X} \mu_q \quad \text{for every subset } X \subset Q ,$$

and *normality*

$$\sum_q \mu_q = 1 \quad (\text{the totality is } 100\%) . \quad (1)$$

The question weights are collected into the column m -vector

$$\boldsymbol{\mu} = \{\mu_q\} .$$

In our application, ‘unweighted’ means equal weights $\mu_q = 1/32$ for the questions covered by opinion polls and $\mu_q = 1/32$ otherwise (to ignore the questions not covered by the polls). The non-normalized expert weights are shown in the third section of Table 2. To fulfill the normalizing condition (1), divide each weight by the total of the weights in the given column.

Candidates Consider N candidates c for election; in our application the candidates are five parties and DGB. Their positive or negative opinions $b_{qc} = \pm 1$ on questions q are collected into the $(m \times N)$ -matrix of candidate opinions derived from the second section of Table 2

$$\mathbf{B} = \{b_{qc}\}, \quad b_{qc} = \pm 1 .$$

Balance of opinions Imagine a *society* of voters for the five leading parties. On each question q , the society falls into *protagonists*, who answer ‘Yes’, and *antagonists*, who answer ‘No’. On every question q , the *balance of opinions* a_q is the predominance of protagonists over antagonists, expressed in fraction of the voters. In our consideration, we assume that the public polls adequately represent the proportion between both groups, so that the balance of opinions a_q is calculated from the normalized balance of opinions

in the polls (shown by the blue bars in Figure 2). For instance, the balance of opinions for question '2. Introduce nation-wide minimal wage' is calculated as follows

$$a_2 = \frac{52 - 43}{52 + 43} = \frac{9}{95} \approx 0.0947 .$$

Popularity and universality of the candidates (parties) The *representativeness* r_{qc} of candidate c on question q is the size of the voters' group represented, measured in fraction (percentage) of the society

$$r_{qc} = \begin{cases} \text{total weight of protagonists in the society} & \text{if } b_{qc} = 1 \\ \text{total weight of antagonists in the society} & \text{if } b_{qc} = -1 \end{cases} .$$

The *popularity* of candidate c is the weighted average of his representativeness (= expected representativeness)

$$P_c = \sum_q \mu_q r_{qc} . \quad (2)$$

The *universality* of candidate c is the weighted frequency with which he represents a non-strict majority (= expected rounded representativeness):

$$U_c = \sum_{q:r_{qc} \geq \frac{1}{2}} \mu_q = \sum_q \mu_q \text{round}[r_{qc}] . \quad (3)$$

In a sense, the popularity reflects the spatial aspect of representativeness, and the universality reflects its temporal aspect.

Computing the indicators and their geometric interpretation Introduce the following notation (all vectors are column vectors!):

' the operation of vector/matrix transpose

. the operation of element-by-element product of vectors and matrices of the same size, for example, $(1, 2) \cdot (3, 4) = (3, 8)$

.² the operation of element-by-element square of vectors and matrices, for example, $(2, 3) \cdot^2 = (4, 9)$

+ the addition of scalars to matrices or vectors by applying it to all matrix elements, for example, $0.5 + (1, 2) = (1.5, 2.5)$

diag \mathbf{a} the diagonal $(m \times m)$ -matrix with elements of vector \mathbf{a} on its main diagonal

sign \mathbf{a} the m -vector of *majority opinions* derived from the vector \mathbf{a} by applying the sign function to its coordinates

$$\text{sign}a_q = \begin{cases} +1 & \text{if } a_q > 0, & \text{i.e. the majority opinion on question } q \text{ is positive} \\ 0 & \text{if } a_q = 0, & \text{i.e. tie opinion on question } q \\ -1 & \text{if } a_q < 0, & \text{i.e. the majority opinion on question } q \text{ is negative} \end{cases}$$

$\delta_{\mathbf{a}} = 1 - \text{abs}(\text{sign}\mathbf{a})$ the m -vector of *indicators of tie opinion*, with the q th coordinate being 1 if the opinion on question q is tied, and 0 otherwise; we use this vector to express the total weight of questions with a tie opinion

$$\boldsymbol{\mu}'\delta_{\mathbf{a}} = \sum_{q:a_q=0} \mu_q \quad (4)$$

Theorem 1 (Computing the indicators and their geometric interpretation)

$$\underbrace{\mathbf{R}}_{\substack{(m \times N)\text{-matrix of} \\ \text{representativeness} \\ \text{of candidates } c \\ \text{on questions } q}} = \frac{1}{2} + \frac{1}{2} \underbrace{\text{diag}\mathbf{a}}_{\substack{\text{diagonal} \\ (m \times m)\text{-matrix of} \\ \text{balance of opinions}}} \underbrace{\mathbf{B}}_{\substack{(m \times N)\text{-matrix} \\ \text{of opinions} \\ \text{of candidates } c \\ \text{on questions } q}} \quad (5)$$

$$\underbrace{\{\mathbf{P}_c\}}_{\substack{\text{row } N\text{-vector of} \\ \text{popularity of} \\ \text{all candidates } c}} = \boldsymbol{\mu}'\mathbf{R} \quad (6)$$

$$= \frac{1}{2} + \frac{1}{2} \underbrace{(\boldsymbol{\mu} \cdot \mathbf{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of balance} \\ \text{of opinions}}} \underbrace{\mathbf{B}}_{\substack{(m \times N)\text{-matrix} \\ \text{of opinions} \\ \text{of candidates } c \\ \text{on questions } q}} \quad (7)$$

$$\underbrace{\{\mathbf{U}_c\}}_{\substack{\text{row } N\text{-vector of} \\ \text{universality of} \\ \text{all candidates } c}} = \boldsymbol{\mu}'\text{round}[\mathbf{R}] \quad (8)$$

$$= \frac{1}{2} + \frac{1}{2} \underbrace{\boldsymbol{\mu}'\delta_{\mathbf{a}}}_{\substack{\text{total weight of} \\ \text{questions with} \\ \text{tie opinions} \\ \text{(constant scalar} \\ \text{independent of } c)}} + \frac{1}{2} \underbrace{(\boldsymbol{\mu} \cdot \text{sign}\mathbf{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of majority} \\ \text{opinion}}} \underbrace{\mathbf{B}}_{\substack{(m \times N)\text{-matrix} \\ \text{of opinions} \\ \text{of candidates } c \\ \text{on questions } q}}, \quad (9)$$

where \mathbf{a} is the vector of balance of opinions.

Thus, the most popular (universal) candidate c has the largest projection of his opinion vector \mathbf{b}_c (= the c th column of matrix \mathbf{B}) on the $\boldsymbol{\mu}$ -weighted vector of balance of opinions $\boldsymbol{\mu} \cdot \mathbf{a}$ (respectively, on the $\boldsymbol{\mu}$ -weighted vector of majority opinion $\boldsymbol{\mu} \cdot \text{sign}\mathbf{a}$).

The formulas of the theorem are used to compute the indicators for Figure 3.

Remark 1 (Analogy with force vectors in physics)

Recall that in mechanics a work is produced by displacements. Accordingly, the only productive constituent of a force vector is its projection on the direction of motion. In Theorem 1, the “work for the society” of a candidate is measured by the projection of his opinion vector on the “main stream”, the vector of balance of opinions in case of popularity, or on the vector of majority opinion in case of universality. Thus all the parties with various opinions are projected onto the ‘vector of political trend’, exactly like in the case of physical forces.

Evaluation of coalitions By definition, a *coalition* C is a subset of the set of candidates. The *coalition weight* is the total percentage of voters for all the candidates of the coalition. For instance, the weight of coalition $C = \text{CDU/CSU/SPD}$ is $33.8 + 23.0 = 56.8\%$.

The (relative) weights of members of coalition C are collected in the normalized vector

$$\overset{C}{\boldsymbol{\xi}} = \left\{ \overset{C}{\xi}_c = \frac{\xi_c}{\sum_{c \in C} \xi_c}, \quad c \in C \right\} .$$

The matrix of opinions of coalition members is the restriction of \mathbf{B} to columns $c \in C$:

$$\overset{C}{\mathbf{B}} = \{b_{qc}, \quad c \in C\} . \quad (10)$$

The *balance of opinions within coalition* C is the vector

$$\overset{C}{\mathbf{b}} = \left\{ \overset{C}{b}_q \right\} = \overset{C}{\mathbf{B}} \overset{C}{\boldsymbol{\xi}} . \quad (11)$$

The (degree of) *unanimity* of coalition C is the total weight of the questions on which the coalition members have equal positions. For instance, SPD and CDU agree in 13 of 32 questions, so that its unanimity is 40.6%.

If a coalition C is unanimous on question q then its representativeness r_{qC} is equal to that of its every member. If the coalition is not unanimous, it can have either positive, or negative opinion on question q . We assume that on question q a coalition C represents protagonists with a probability ranging from the relative weight $\overset{C}{\xi}_q^+$ of coalition protagonists to the absolute uncertainty $\frac{1}{2}$:

$$p \overset{C}{\xi}_q^+ + (1-p) \frac{1}{2}, \quad 0 \leq p \leq 1 ,$$

where p denotes the *proportionality of impact to weights* of coalition members. If $p = 1$ the impact of coalition members is proportional to their weights. If $p = 0$ the coalition has Yes/No opinions with equal chances, so that both protagonists and antagonists are represented with equal probabilities $\frac{1}{2}$.

Under these provisions, the representativeness and the indicators of popularity and universality of a coalition are random variables which behavior for non-unanimous questions depends on the parameter p . The *popularity* \mathbf{P}_C and *universality* \mathbf{U}_C of coalition C are understood as its expected representativeness and expected rounded representativeness. Besides, we compute the variance of representativeness and of rounded representativeness as a measure of accuracy of the coalition indicators.

Theorem 2 (Evaluation of coalitions)

$$\text{Unanimity of } C = 1 - \boldsymbol{\mu}' \overset{C}{\mathbf{s}} \quad (12)$$

$$\begin{aligned} \mathbf{P}_C = \mathbf{E} r_C &= \underbrace{\overline{\mathbf{P}_C}}_{\substack{\text{weighted} \\ \text{average} \\ \text{popularity} \\ \text{of coalition} \\ \text{members}}} - \frac{1}{2} \underbrace{(1-p)}_{\text{impact of uncertainty}} \underbrace{(\boldsymbol{\mu} \cdot \mathbf{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of balance} \\ \text{of opinions}}} \underbrace{\begin{pmatrix} \overset{C}{\mathbf{s}} \\ \overset{C}{\mathbf{b}} \end{pmatrix}}_{\substack{m\text{-vector} \\ \text{of balance} \\ \text{of opinions} \\ \text{within the} \\ \text{coalition for} \\ \text{non-unanimous} \\ \text{questions}}} \end{aligned} \quad (13)$$

$$\begin{aligned}
U_C = E \text{ round}[r_C] &= \underbrace{\overline{U_C}}_{\substack{= \sum_{c \in C} \xi_c^C U_c \\ \text{weighted} \\ \text{average} \\ \text{universality} \\ \text{of coalition} \\ \text{members}}} - \frac{1}{2} \underbrace{(1-p)}_{\text{impact of uncertainty}} \underbrace{(\boldsymbol{\mu} \cdot \text{sign} \mathbf{a})'}_{\substack{\boldsymbol{\mu}\text{-weighted} \\ m\text{-vector} \\ \text{of majority} \\ \text{opinion}}} \underbrace{\begin{pmatrix} C \\ \mathbf{s} \cdot \mathbf{b} \end{pmatrix}}_{\substack{m\text{-vector} \\ \text{of balance} \\ \text{of opinions} \\ \text{within the} \\ \text{coalition for} \\ \text{non-unanimous} \\ \text{questions}}}, \quad (14)
\end{aligned}$$

where

$$\mathbf{s} = \left\{ s_q^C = \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) \right\}, \quad \text{are indicators of the coalition non-unanimity on questions } q, \text{ with } n \text{ being the number of coalition members,}$$

$$\overline{P_C} = \sum_{c \in C} \xi_c^C P_c \quad \text{is the weighted average popularity of coalition members, and}$$

$$\overline{U_C} = \sum_{c \in C} \xi_c^C U_c \quad \text{is the weighted average universality of coalition members.}$$

Besides, if the coalition opinions on non-unanimous questions are independent (= independent negotiations on every question) then

$$V r_C = \frac{1}{4} [(\boldsymbol{\mu} \cdot \mathbf{a}) \cdot^2]' \left[\begin{matrix} C \\ \mathbf{s} \cdot \left(1 - p^2 \mathbf{b} \cdot^2 \right) \end{matrix} \right] \quad (15)$$

$$V \text{ round}[r_C] = \frac{1}{4} [(\boldsymbol{\mu} \cdot \text{sign} \mathbf{a}) \cdot^2]' \left[\begin{matrix} C \\ \mathbf{s} \cdot \left(1 - p^2 \mathbf{b} \cdot^2 \right) \end{matrix} \right]. \quad (16)$$

The formulas of the theorem with $p = \frac{1}{2}$ are used to compute indicators in Table 4 and for Figures 4 and 5.

Remark 2 (Coalition indicators in the simplest case)

If $p = 1$ (the impact of coalition members is proportional to their weights) then by (13) and (14) the popularity and universality of a coalition are equal to the weighted average of corresponding indicators of its members: $P_C = \overline{P_C}$ and $U_C = \overline{U_C}$.

Proof of Theorem 1

On every question q , obviously

$$\begin{aligned}
\text{The weight of non-strict majority/minority} &= \underbrace{\frac{1}{2}}_{\substack{\text{half} \\ \text{the} \\ \text{society}}} \pm \frac{1}{2} \underbrace{|a_q|}_{\substack{\text{predominance} \\ \text{of protagonists} \\ \text{over antagonists}}}.
\end{aligned}$$

Consequently, the representativeness of candidate c on question q is determined by the the sign of his opinion and by the sign of a_q

$$\begin{aligned}
r_{qc} &= \underbrace{\frac{1}{2}}_{\substack{\text{half} \\ \text{the} \\ \text{society}}} + \frac{1}{2} \underbrace{a_q}_{\substack{\text{predominance} \\ \text{of protagonists} \\ \text{over antagonists} \\ \text{in the society}}} \underbrace{b_{qc}}_{\substack{= \pm 1 \\ \text{opinion of} \\ \text{candidate } c}}, \quad (17)
\end{aligned}$$

which matrix form is

$$\mathbf{R} = \frac{1}{2} + \frac{1}{2} \text{diag} \mathbf{a} \mathbf{B} , \quad (18)$$

as required in (5). Multiplying $\boldsymbol{\mu}'$ by (18), as required in the definition (2), obtain (6) and (7):

$$\begin{aligned} \{\mathbf{P}_c\} &= \boldsymbol{\mu}' \mathbf{R} \\ &= \boldsymbol{\mu}' \left[\frac{1}{2} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + \frac{1}{2} \text{diag} \mathbf{a} \mathbf{B} \right] \\ &= \frac{1}{2} \cdot \underbrace{\sum_q \mu_q}_{=1} + \frac{1}{2} \boldsymbol{\mu}' \text{diag} \mathbf{a} \mathbf{B} \\ &= \frac{1}{2} + \frac{1}{2} (\boldsymbol{\mu} \cdot \mathbf{a})' \mathbf{B} . \end{aligned} \quad (19)$$

To obtain (8), multiply $\boldsymbol{\mu}'$ by $\text{round}[\mathbf{R}]$, as required by definition (3). To obtain (9), express the rounded representativeness of candidate c on question q by analogy with (17):

$$\begin{aligned} \text{round}[r_{qc}] &= \frac{1}{2} + \frac{1}{2} \text{sign} a_q b_{qc} + \frac{1}{2} \underbrace{\delta_{a_q}}_{\substack{= 1 & \text{if } a_q = 0 \\ = 0 & \text{if } a_q \neq 0}} \end{aligned} \quad (20)$$

and proceed similarly to (19). ■

Proof of Theorem 2

The unanimity of coalition members $c \in C$ on question q means that either all $b_{qc} = 1$, or all $b_{qc} = -1$, implying $|\sum_{c \in C} b_{cq}| = n$, where n is the number of coalition members. Consequently,

$$\overset{C}{s}_q = \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) = \begin{cases} 0 & \text{if } c \in C \text{ are unanimous on question } q \\ 1 & \text{otherwise} \end{cases} \quad (21)$$

$$1 - \overset{C}{s}_q = 1 - \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) = \begin{cases} 1 & \text{if } c \in C \text{ are unanimous on question } q \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

Hence, the total weight of the questions on which the coalition is unanimous

$$\begin{aligned} \text{Unanimity of } C &= \sum_q \mu_q \left[1 - \text{sign} \left(n - \left| \sum_{c \in C} b_{cq} \right| \right) \right] \\ &= \sum_q \underbrace{\mu_q}_{=1} - \boldsymbol{\mu}' \overset{C}{\mathbf{s}} , \end{aligned}$$

as required in (12).

Compute the expectation and variance of representativeness r_{qC} of a coalition C for a given question q . Consider two cases.

- **Coalition members are unanimous on question q .** Express the coalition's representativeness by analogy with (17) and note that r_{qC} is constant, implying

$$\mathbb{E} r_{qC} = \frac{1}{2} + \frac{1}{2} a_q \underbrace{\overset{C}{b_q}}_{=\pm 1 \text{ in case of unanimity}} \quad (23)$$

$$\mathbb{V} r_{qC} = 0 . \quad (24)$$

- **Coalition members are not unanimous on question q .** The representativeness r_{qC} is a Bernoulli random variable, taking two values $\frac{1}{2} \pm \frac{1}{2} a_q$ with range $|a_q|$. The coalition can accept the opinion of majority in the coalition, or of its minority, representing respectively the social groups with the size:

$$\frac{1}{2} + \frac{1}{2} a_q \text{sign } \overset{C}{b_q} \quad \text{with probability} \quad \underbrace{\left(\frac{1}{2} + \frac{1}{2} \left| \overset{C}{b_q} \right| \right)}_{\text{majority in the coalition}} p + \frac{1}{2} (1-p) \quad (25)$$

$$= \frac{1}{2} + \frac{1}{2} \left| \overset{C}{b_q} \right| p ,$$

$$\frac{1}{2} - \frac{1}{2} a_q \text{sign } \overset{C}{b_q} \quad \text{with probability} \quad \frac{1}{2} - \frac{1}{2} \left| \overset{C}{b_q} \right| p . \quad (26)$$

By definition of expectation

$$\begin{aligned} \mathbb{E} r_{qC} &= \frac{1}{4} \left[\left(1 + a_q \text{sign } \overset{C}{b_q} \right) \left(1 + \left| \overset{C}{b_q} \right| p \right) + \left(1 - a_q \text{sign } \overset{C}{b_q} \right) \left(1 - \left| \overset{C}{b_q} \right| p \right) \right] \\ &= \frac{1}{2} + \frac{1}{2} a_q \text{sign } \overset{C}{b_q} \left| \overset{C}{b_q} \right| p \\ &= \frac{1}{2} + \frac{1}{2} a_q \overset{C}{b_q} p . \end{aligned} \quad (27)$$

By the known formula for the variance of Bernoulli random variables (Korn and Korn 1968, Table 18.8.3, case $n = 1$, Abramowitz and Stegun 1972, 26.1.20, case $n = 1$) obtain

$$\begin{aligned} \mathbb{V} r_{qC} &= a_q^2 \left(\frac{1}{2} + \frac{1}{2} \left| \overset{C}{b_q} \right| p \right) \left(\frac{1}{2} - \frac{1}{2} \left| \overset{C}{b_q} \right| p \right) \\ &= \frac{1}{4} a_q^2 \left[1 - \left(\overset{C}{b_q} \right)^2 p^2 \right] . \end{aligned} \quad (28)$$

Compute the popularity P_C of a coalition C . Using (23) and (27) and applying (21)–(22) as indicators of (non-) unanimity, obtain

$$\begin{aligned} P_C = \mathbb{E} r_C &= \sum_{q \text{ unanimous}} \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q \overset{C}{b_q} \right) + \sum_{q \text{ non-unanimous}} \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q \overset{C}{b_q} p \right) \\ &= \sum_q \mu_q \left(1 - \overset{C}{s_q} \right) \left(\frac{1}{2} + \frac{1}{2} a_q \overset{C}{b_q} \right) + \sum_q \mu_q \overset{C}{s_q} \left(\frac{1}{2} + \frac{1}{2} a_q \overset{C}{b_q} p \right) \quad \xRightarrow{\text{Identity}} \end{aligned}$$

$$\begin{aligned}
&= \sum_q \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q \overset{C}{b}_q \right) - \frac{1}{2} (1-p) \sum_q \mu_q a_q \overset{C}{s}_q \overset{C}{b}_q \stackrel{\text{by (10)}}{\implies} \quad (29) \\
&= \underbrace{\sum_q \mu_q \left(\frac{1}{2} \sum_{c \in C} \overset{C}{\xi}_c + \frac{1}{2} a_q \sum_{c \in C} \overset{C}{\xi}_c \overset{C}{b}_{qc} \right)}_{=1} - \frac{1}{2} (1-p) \sum_q \mu_q a_q \overset{C}{s}_q \overset{C}{b}_q \\
&\quad \sum_{c \in C} \overset{C}{\xi}_c \sum_q \mu_q \left(\frac{1}{2} + \frac{1}{2} a_q \overset{C}{b}_{qc} \right) \stackrel{\text{by (2) and (17)}}{=} \sum_{c \in C} \overset{C}{\xi}_c \mathbf{P}_c \\
&= \underbrace{\sum_{c \in C} \overset{C}{\xi}_c \mathbf{P}_c}_{\overline{\mathbf{P}}_C} - \frac{1}{2} (1-p) (\boldsymbol{\mu} \cdot \mathbf{a})' \left(\overset{C}{\mathbf{s}} \cdot \overset{C}{\mathbf{b}} \right) ,
\end{aligned}$$

as required in (13).

Compute the universality \mathbf{U}_C of a coalition C . If $a_q = 0$ (tie opinion on question q in the society) the rounded representativeness $\text{round}[r_C] = \text{round}[\frac{1}{2}] = \frac{1}{2}$. If $a_q \neq 0$, by analogy with (25)–(26), the rounded representativeness of coalition C takes values

$$\begin{aligned}
\frac{1}{2} + \frac{1}{2} \text{sign} a_q \text{sign} \overset{C}{b}_q &\quad \text{with probability} \quad \frac{1}{2} + \frac{1}{2} \left| \overset{C}{b}_q \right| p , \\
\frac{1}{2} - \frac{1}{2} \text{sign} a_q \text{sign} \overset{C}{b}_q &\quad \text{with probability} \quad \frac{1}{2} - \frac{1}{2} \left| \overset{C}{b}_q \right| p .
\end{aligned}$$

Applying the indicator of tie opinion (4) and proceeding like in (27) and (29) obtain

$$\begin{aligned}
\mathbf{U}_C &= \mathbf{E} \text{round}[r_C] \\
&= \underbrace{\sum_{q: a_q=0} \mu_q \cdot 1 + \sum_{q: a_q \neq 0} \mu_q \left(\frac{1}{2} + \frac{1}{2} \text{sign} a_q \overset{C}{b}_q \right)}_{\sum_{c \in C} \overset{C}{\xi}_c \mathbf{U}_c} - \frac{1}{2} (1-p) \underbrace{\sum_{q: a_q \neq 0} \mu_q \text{sign} a_q \overset{C}{s}_q \overset{C}{b}_q}_{\sum_q \mu_q \text{sign} a_q \overset{C}{s}_q \overset{C}{b}_q} \\
&= \overline{\mathbf{U}}_C - \frac{1}{2} (1-p) (\boldsymbol{\mu} \cdot \text{sign} \mathbf{a})' \left(\overset{C}{\mathbf{s}} \cdot \overset{C}{\mathbf{b}} \right) ,
\end{aligned}$$

as required in (14).

Compute the variance of representativeness r_{qC} of coalition C . Using (24) and (28), applying (21)–(22) as indicators of (non-)unanimity, and taking into account that the variance of a sum of independent random variables is the sum of their variances, obtain

$$\begin{aligned}
\mathbf{V} r_C &= \sum_q \mu_q^2 \left(1 - \overset{C}{s}_q \right) \cdot 0 + \sum_q \mu_q^2 \overset{C}{s}_q \frac{1}{4} a_q^2 \left[1 - p^2 \left(\overset{C}{b}_q \right)^2 \right] \stackrel{\text{Identity}}{\implies} \quad (30) \\
&= \frac{1}{4} [(\boldsymbol{\mu} \cdot \mathbf{a}) \cdot]' \left[\overset{C}{\mathbf{s}} \cdot \left(1 - p^2 \overset{C}{\mathbf{b}} \cdot \right) \right] ,
\end{aligned}$$

as required in (15).

The derivation of the variance for the rounded representativeness $\text{round}[r_{qC}]$ of coalition C is similar to (30), but there are two changes to be made:

- The range of the Bernoulli random variable $\text{round}[r_{qC}]$ is 1 instead of $|a_q|$, consequently, a_q^2 in (30) should be replaced by 1.

- If $a_q = 0$ in (30), that is, tie opinion in the society on question q , then $r_{qC} = \frac{1}{2}$, implying $\text{round}[r_{qC}] = \text{round}[\frac{1}{2}] = 1$. Hence, $\text{V round}[r_{qC}] = 0$. The variances for such questions q should be nullified by the multiplier $(\text{sign}a_q)^2$ which retains all other terms intact.

Thus,

$$\text{V round}[r_C] = \frac{1}{4} [\boldsymbol{\mu} \cdot (\text{sign}\mathbf{a}) \cdot \mathbf{1}]' \left[\mathbf{s} \cdot \left(1 - p^2 \mathbf{b} \cdot \mathbf{1} \right) \right] ,$$

as required in (16). ■

8 Annex 2. Polls of public opinion

The references to polls of public opinions used in this paper are collected in Table 10.

The Wahl-O-Mat question number and the question (shortly formulated)	Description of survey data		
	Survey date	Sources (all accessed on Nov 11 2009)	Comments
Labour market			
2. Introduce nation-wide minimal wage	Jan 08	http://www.forschungsgruppe.de/Umfragen_und_Publikationen/Politbarometer/Archiv/Politbarometer.2008/Januar.2008/	tio-
17. Relax protection against dismissals	May 08	http://www.boeckler.de/32015_92020.html	
Economy and taxes			
24. Exclusive governmental ownership of railways	Mar 08	http://privatisierungstoppen.deinebahn.de/download/umfrage-bahnprivatisierung_08-03-27.pdf	emndid-
10. Equity holding by government in private banks has to be temporary	Oct 08	http://de.statista.com/statistik/daten/studie/2065/umfrage/einfluss-des-staates-auf-privatbanken/#stat	not exact question, different surveys
5. No state control over top-management salaries*	Mar 09	http://www.forschungsgruppe.de/Umfragen_und_Publikationen/Politbarometer/Archiv/Politbarometer.2009/Maerz.I/	tio-
13. Decrease corporate taxes	Oct 09	http://www.infratest-dimap.de/umfragen-analysen/bundesweit/ard-deutschlandtrend/2009/oktober/	
28. Reintroduce a wealth tax*	Dec 07	Bertelsmann Stiftung, "Soziale Gerechtigkeit 2007 - Ergebnisse einer repräsentativen Bürgerumfrage", http://www.bertelsmannstiftung.de/bst/de/media/xcms_bst_dms_23333_23334_2.pdf	not exactly dichotomous questions
Environment			
1. Prolong the operation time of nuclear power plants	Jul 09	http://www.forschungsgruppe.de/Umfragen_und_Publikationen/Politbarometer/Archiv/Politbarometer.2009/Juli.II/	tio-
19. Introduce a general speed limit on Autobahnen (German motorways)	Jul 08	http://de.statista.com/statistik/daten/studie/1362/umfrage/ansicht-zur-einfuehrung-eines-generellen-temperaturlimits/#info	mpolim-
9. Unexceptionally ban experiments on animals	Aug 03	http://www.greenpeace-magazin.de/index.php?id=3359	
26. Authorize production of genetically modified food*	May 09	http://www.slowfood.de/w/files/pdf_neu/meinungen_zu_gentechnik_190509.pdf	
22. More subsidies for eco-farming		x	

* Adjustments to non-specified party positions based on public statements, parliament voting, etc.

The Wahl-O-Mat question number and the question (shortly formulated)	Description of survey data		
	Survey date	Sources (all accessed on Nov 11 2009)	Comments
Social policy			
36. Increase significantly unemployment benefits (Hartz IV)*	Oct 09	http://de.statista.com/statistik/daten/studie/70305/umfrage/einschaetzung-der-hoehe-der-hartz-iv-saetze/#info	
32. If wages decrease, pensions can be reduced*	May 09	http://www.infratest-dimap.de/umfragen-analysen/bundesweit/ard-deutschlandtrend/2009/mai/	
31. No Praxisgebuehr (quarterly fee for medical visits)	Feb 04	http://www.infratest-dimap.de/de/umfragen-analysen/bundesweit/ard-deutschlandtrend/2004/februar/	
15. Compensation to parents who use no public daycares	Nov 09	http://www.focus.de/politik/deutschland/regierungspbruederle-haelt-nichts-von-betreuungsgeld_aid_450166.html	laene- no detailed numbers
Education			
29. Leave the education policy under the authority of the states	Aug 09	http://www.eltern.de/familie-und-urlaub/familienpolitik/bundestagswahl-familie.html?page=9	only survey among parents
34. Leave 3 types of schools with different access to further education	Sep 09	http://www.dphv.de/fileadmin/user_upload/news/infothek/2009/FORSA-Umfrage_Einstellungen_zur_Einheitsschule_grafik.pdf	
16. Guarantee an apprenticeship training position for every adolescent*	Dec 07	Bertelsmann Stiftung, "Soziale Gerechtigkeit 2007 - Ergebnisse einer repräsentativen Bürgerumfrage", http://www.bertelsmannstiftung.de/bst/de/media/xcms_bst_dms_23333_23334_2.pdf	not exactly dichotomous questions
7. The first university degree should be free of tuition fees*	Dec 07	Bertelsmann Stiftung, "Soziale Gerechtigkeit 2007 - Ergebnisse einer repräsentativen Bürgerumfrage", http://www.bertelsmannstiftung.de/bst/de/media/xcms_bst_dms_23333_23334_2.pdf , p. 12	not exactly dichotomous questions (percentage of the positive answer to question "Abolish fees for studies")
8. Obligatory language test for all children of preschool age		x	
20. BAFOeG (aid to students and trainees) regardless of parent income		x	
Gender			
12. There should be a quota for women in leading positions	Mar 09	https://www.berlin.de/imperia/md/content/sen-genderquote.pdf?start&ts=1244898971	/frauen-
27. Full adoption rights for homosexual couples	Jul 09	http://www.welt.de/politik/deutschland/article41768fordert-Adoptionsrecht-fuer-Homosexuelle.html	30/Zypries- non-scientific online survey with 16000 responses

Table 10: Sheet B. Data sources

* Adjustments to non-specified party positions based on public statements, parliament voting, etc.

The Wahl-O-Mat question number and the question (shortly formulated)	Description of survey data		
	Survey date	Sources (all accessed on Nov 11 2009)	Comments
Domestic policy			
6. Prohibit secret online surveillance of private computers	Nov 08	http://www.forschungsgruppe.de/Umfragen_und_Publikationen/Politbarometer/Archiv/Politbarometer.2008/November_II_2008/	tio- vem-
25. Retain the compulsory military service	Oct 06	http://www.focus.de/politik/deutschland/umfrage_aid_117913.html	
37. Allow domestic use of German military forces against terrorism	Oct 08	http://www.forschungsgruppe.de/Umfragen_und_Publikationen/Politbarometer/Archiv/Politbarometer.2008/Oktober_I_2008/	tio-
14. Introduce referenda at the federal level	Jun 09	http://www.mehr-demokratie.de/presse-hintergrund.html	
23. German politics should follow Christian values*	Aug 03	http://www.kas.de/wf/doc/kas.1870-544-1-30.pdf	33 % for more influence, 20% for less influence, 40% accept the current situation
38. The German democracy is the best form of government	May 09	http://www.forschungsgruppewahlen.de/Umfragen_und_Publikationen/Archiv_weitere_Umfragen/Demokratie_und_Integration_1/	
35. Municipal voting rights for foreign permanent residents	Jul 06	http://de.statista.com/statistik/diagramm/studie/40104/umfrage/kommunales-wahlrecht-fuer-alle-auslaender-in-deutschland/#info	er-in-
30. Less restriction on asylum policy		x	
Foreign policy			
3. Immediate withdrawal of German troops from Afghanistan	Sep 09	http://www.infratest-dimap.de/umfragen-analysen/bundesweit/ard-deutschlandtrend/2009/september-extra/	x
11. No trade relations with governments who violate human rights		x	
33. General export prohibition of military materials		x	
European policy			
4. Germany should leave the European Union	Oct 08	http://www.bertelsmann-stiftung.de/cps/rde/xchg/bst/hs.xsl/nachrichten_91928.htm	
18. Turkey should be a full member of the European Union	Jun 09	http://www.rp-online.de/politik/europawahl/Schulz-S-fuer-Tuerkei-Beitritt-zur-EU_aid_716658.html	PD-wirbt-

* Adjustments to non-specified party positions based on public statements, parliament voting, etc.

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