

Impressum

Karlsruher Institut für Technologie (KIT)
Fakultät für Wirtschaftswissenschaften
Institut für Volkswirtschaftslehre (ECON)

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KIT – Universität des Landes Baden-Württemberg und
nationales Forschungszentrum in der Helmholtz-Gemeinschaft

Working Paper Series in Economics
No. 66, April 2015

ISSN 2190-9806

econpapers.wiwi.kit.edu

Institut für Wirtschaftstheorie und Operations Research
Karlsruhe Institute of Technology

**Decision making in politics and economics:
6. Empirically Constructing the German Political Spectrum**

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Working paper Nr. 66

April 2015

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Abstract

The advocates of modern western democracy promote the viewpoint that the class division of the society is becoming outdated. We attempt to disprove this statement with an example of 28 German parties who participated in the 2013 federal election. The official party positions on 38 policy issues are considered and the parties are identified with vectors of this 38-dimensional policy space. The statement in question, that there is no predominant political axis, would imply that the party vectors are scattered homogeneously, making a ball-shaped cloud of ‘observations’. However, the Prime Component Analysis (PCA) shows that the party vectors constitute a thin ellipsoid whose two longest diameters cover 83.4% of the total variance. The consequent party ordering is the left-right axis rolled in a circumference, making the far-left and far-right ends meet. Basing on this empirical evidence, we conclude that neither the left–right characterization of parties nor the class opposition is outdated.

Next, it is shown that the electoral success is highly correlated with the number of party members, but not with the party’s capacity to represent public opinion. For this purpose, a representativeness index is defined which measures how well the party policy profiles match with the results of 36 public opinion polls on 36 out of the 38 policy issues mentioned. To reveal representativeness trends, the parties are ordered contiguously, with neighboring parties having close policy profiles. This contiguous ordering is found with four optimization methods: (1) dimensionality reduction by means of PCA, (2) traveling salesman problem to construct the shortest chain of proximate parties, (3) least squares to minimize the distances between parties with close profiles, and (4) largest squares to maximize the distances between parties with opposite profiles. The most salient trend is observed for the circular left–right party ordering found with the PCA. The best representatives of public opinion are the moderate left, next come the far-left and the far-right, and the least representative are moderate right (conservative) parties.

All of these imply the following warning. Since the collapse of communism damaged significantly the image of the left, their election today looks hardly probable, but the power can be taken by the next-representative far-right parties who already represent public opinion better than the currently governing conservative party.

Contents

1	Introduction	7
2	The model	10
3	Principal Component Analysis Solution	16
4	Traveling Salesman Problem Solution	20
5	Weighted least squares solution	23
6	Weighted largest squares solution	23
7	Conclusions	26
8	References	27

1 Introduction

Following [Marx 1867, Weber 1921], economists and sociologists consider classes as social groups with common interests determined by income, property, education, social status, and relation to the means of production. Their competing interests result in the class struggle headed by ‘left’ and ‘right’ political parties that emerged after the industrial revolution. The left (labourists, social-democrats and communists) stand for egalitarianism, solidarity with income redistribution, and governmental intervention in the economy. The right (conservatives and economic liberals) defend private property, free entrepreneurship, and equal opportunities. Until recently the class struggle has been regarded as the major political drive, and, correspondingly, the left-right axis has been predominantly used to locate political agents in the political space [Blattberg 2009, Bobbio and Cameron 1996, Gauchet 1996, Left-right politics 2015, Lipset 1960, Knapp and Wright 2001, Mahoney, Coogle and Banks 1984, Political spectrum 2015, Rous and Lee 1978, Ruypers 2005, Ware 1996, Wilson 2004].

Discussing radical changes in the world order at the turn of the century, the advocates of modern western democracy promote the viewpoint that the class division of the society is becoming outdated; see for instance [Giddens 1994, Manin 1997, Mitchell 2007, Sulakshin 2010, Voda 2014]. It is argued that after the Soviet Union and Eastern Block ceased to exist, the class struggle lost its inspiration by a systemic alternative. On the other hand, climate change, globalization, competition of the West with inexorably rising China and India, aging population, migration, ethnic tensions, religious intolerance, and international terrorism have swayed the public attention away from left-right political confrontations toward less ideological and more pragmatic matters. For instance, subordinating international class interests to national geopolitical challenges, [Streeck 1999] develops the idea of employer-employee ‘competitive solidarity’, which to a certain extent supplants that of class struggle. Some authors emphasize that due to increasing interdependence between countries, political platforms have come to be perceived as a constraint for flexibly responding to the globalization trends. This results in the emergence of less platform-determined, manager-type politicians who compete for votes by adjusting their positions to numerous cleavages of the society and advertising themselves in the media before large audiences:

In party democracy electoral cleavages reflect class division. In a number of Western societies the situation today is different. No socioeconomic or cultural cleavage is evidently more important and stable than others. To be sure, citizens do not constitute a homogeneous mass that can be divided in any manner by the choices they are offered, but the social and cultural lines of cleavage are numerous, crosscutting, and rapidly changing. Such an electorate is capable of a number of splits. The number of floating voters who do not cast their ballot on the basis of stable party identification is increasing. A growing segment of the electorate tends to vote according to the stakes and issues of each election.

[Manin 1997, pp. 209, 223, 231].

From all of these, it is concluded that the political spectrum is becoming essentially multi-dimensional, replacing the former left-right ideological alignment. This viewpoint is reflected in numerous studies, particularly referring to the prize-winning MANIFESTO-database with up to over 400-dimensional representation of party programs from more than 50 countries covering all free democratic elections since 1945 [Budge et al 2001, Budge and McDonald 2007, Klingemann et al 2006, Linhart and Shikano 2009, Volkens et al 2013, WZB 2015]. The internet voting advice applications (VAAs) implemented in about 20 countries also assume multiple cleavages and, correspondingly, multidimensional political spectra [EU profiler 2009, Garzia and Marschall 2014, Kieskompas 2006, Vote match Europe 2015].

We attempt to disprove the statement about multiplicity of equally important political dimensions with an example of German political space represented by 28 political parties who participated in the 2013 Bundestag (federal) election. We consider the official party positions on 38 topical issues declared shortly before the election [Bundeszentrale für politische Bildung 2013] and associate the parties with vectors of their policy profiles in the corresponding 38-dimensional political space. The statement in question, that the left-right axis is no longer predominant, would imply that the party vectors should be scattered more or less homogeneously, resulting in a ball-shaped cloud of ‘observations’. However, the Prime Component Analysis (PCA) reveals that the party vectors actually constitute a thin ellipsoid, whose two longest diameters explain 83.4% of the total variance. The consequent party ordering is the left-right alignment.

However, this result is not that straightforward. It turns out that the left-right axis is rolled into a circumference, reflecting the fact that the far-left and far-right ends meet. This explains why some empirical models fail to recognize a one-dimensional political spectrum [Sulakshin 2010, Voda 2014]: a circumference, being one-dimensional itself, cannot be placed in a one-dimensional Euclidian space — to be accommodated it needs a Euclidian space with at least two line axes. Thereby, our finding bridges two types of spatial political models [Gill and Hangartner 2010, Sect. 8]: directional models of successive policy shifts with circular representations and angular measures [Grofman 1985, Linhart and Shikano 2009, Matthews 1979, Rabinowitz and MacDonald 1989, Schofield 1985], and proximity models, which describe the distance between political agents in the Euclidian space with line axes.

The form of German political spectrum found is further confirmed by a clear trend in the party representativeness along the circular left-right axis. This logic of this implication is as follows. If the left-right alignment were outdated, the party capacity to represent public opinion would not depend on its left-right orientation but on some other factors regarded as more important. The latter is disproved by showing that to a great extent the party representativeness depends just on its left-right orientation. For this purpose, we define a representativeness index, which measures how well the party positions match with the outcomes of public opinion polls on the policy issues considered. Then we try to recognize statistically significant trends in this index with respect to alternative party orderings. Salient trends are observed when the parties are located along the left-right political axis, and the circular model exhibit even better results.

It turns out that the representativeness index exhibits no trend when the parties are ordered

by votes received, as standard in electoral reports. The party's number of votes highly correlates with the number of party members but negatively correlates with the party's representativeness, though insignificantly. On the other hand, a clear trend emerges when the parties are contiguously ordered with regard to the closeness of their political profiles. To find such an ordering, four optimization methods are applied: (1) dimensionality reduction by means of PCA, (2) traveling salesman problem to construct the shortest chain of proximate parties, (3) least squares to minimize the distances between parties with close profiles, and (4) largest squares to maximize the distances between parties with opposite profiles. The most convincing result with a clear representativeness trend is observed for the circular left–right party ordering found with the PCA. Generally, the highest representativeness is inherent in the left-hand end of the political spectrum, and the lowest in the right-hand one. The circular model introduces a further refinement: the best representatives of public opinion are the moderate left, next come the far-left and the far-right, and the least representative are moderate right (conservative) parties.

Basing on this empirical evidence, we conclude that the left–right characterization of parties which reflects the class opposition remains valid, being in no case outdated. Since the collapse of communism damaged significantly the image of the left, their election today looks hardly probable. At the same time, our study indicates at the raising far-right wing, whose representativeness already surpasses that of the conservative and centrist parties that has won the 2013 election. However, it looks that the superior representativeness of the moderate left can help to restore their influence and presence in politics, decreasing that of the far-right.

In Section 2, ‘The model’, the data structure and the data derivatives for the model are introduced. It is shown that the standard party ordering by votes received in election exhibits no statistical significant trend in the party's capacity to represent public opinion.

In Section 3, ‘Principal Component Analysis Solution’, a contiguous party ordering with a salient trend in the party's capacity to represent public opinion is obtained obtained by the model dimensionality reduction.

In Section 4, ‘Traveling Salesman Problem Solution’, the task is reformulated in terms of destinations and distances, and a contiguous party ordering desired is obtained by minimizing the itinerary through all the destinations.

In Section 5, ‘Weighted least squares solution’, a contiguous party ordering is obtained by minimizing the total weighted squared distance of the cells of the correlation triangle to its diagonal weighted with the corresponding correlation coefficients with the opposite sign.

In Section 6, ‘Weighted largest squares solution’, a contiguous party ordering is obtained by maximizing the total weighted squared distance of the cells of the correlation triangle from its bottom-left edge weighted with the corresponding correlation coefficients with the opposite sign.

In Section 7, ‘Conclusions’, the results of the paper are recapitulated and put in the context.

2 The model

The outcomes of the 2013 German Bundestag (federal) election for the 28 participating parties are shown in Table 1.¹ As usual in electoral reports, the parties are ordered by decreasing number of votes received. However, this information is insufficient to analyze the German political spectrum, and we also consider the data in Table 2 with 38 policy questions, estimates of their importance (weights), and balances of public opinion and party positions on these questions. The questions and the party positions are taken from the *Wahl-O-Mat* — voting advice application of the German Federal Agency for Civic Education [Bundeszentrale für politische Bildung 2013]. Recall that the *Wahl-O-Mat* (an invented word composed from the German *Wahl* = election and *Automat*) is the German version of the Dutch Internet site *StemWijzer* (‘VoteMatch’), which was originally developed in the 1990s to involve young people in political participation [Pro demos 2014]. Both web sites help users locate themselves on the political landscape by testing how well their opinions match with party positions. Before an election (local, regional, federal, and European), a special governmental supervising committee compiles a list of questions on topical policy issues (‘Introduce minimum wage?’-Yes/No, ‘Introduce a general speed limit on motorways?’- Yes/No, etc.) and asks the parties participating in the election for their answers. A user of the site answers the same questions, eventually attributing weights to reflect their importance, and then the program compares his or her political profile with that of the parties and finds the best-matching party, the second best-matching party, etc. To exclude manipulations, neither individual data, nor cumulative statistics are available from the *Wahl-O-Mat*. Even if they were available, they could characterize only the position of internet users rather than of the whole electorate: about 44 Mio Germans took part in the 2013 election, whereas the *Wahl-O-Mat* had about 13 M visitors on this occasion, ca. 30% of the voters [Bundeswahlleiter 2013, Bundeszentrale für politische Bildung 2014]. Therefore, the balances of public opinion in Table 2 are taken from relevant public opinion polls. The importance of the questions is estimated by their weighting in four versions: equal (for ‘unweighted’ questions); \log_2 of thousand Google hits for the question keywords (the logarithm with base 2 is a standard device to transform linear measures into perception scales); and two expert scores, both ranging from 0 (unimportant) to 3 (very important) — by the director of the Institute of Economic and Social Research (WSI), Dsseldorf, Professor Brigitte Unger, and the Editor-in-Chief of the info-service *Einblick*, Berlin, Anne Graef.

The bottom line of Table 2 contains the representativeness index of the parties. Firstly, the popularity index — the percentage of the population represented by the party averaged on all the questions — is computed in four versions for the four question weightings. Secondly, the universality index — the percentage of the questions, for which the party represents a majority of the population — is also computed in four versions for the four question weightings. The party’s representativeness index is the mean of these four popularity and four universality

¹All computations, as well as most tables and figures of the paper are made with the MATLAB (version 2014b) programming environment optionally equipped with the MATLAB statistics and optimization toolboxes. The exceptions are the official party logos in Table 1 and the torus in Figure 9 taken from Wikipedia.

indices. The details of the index construction as well as the full information on the party answers, their comments on them, and the description of the public opinion polls with all the references are given in the report [Tangian 2013]; for the general methodology see [Tangian 2014]. As in Tables 1 and 2, the parties in Figure 1 are ordered by decreasing number of votes in the 2013 election). Three curves show the party’s votes received, number of members, and representativeness index. The correlation coefficients at the top of the figure indicate that the party’s electoral success depends on the number of its members but not on its representativeness. Indeed, the votes received and the number of party members are highly correlated (98%), whereas the correlation between the votes received and the party’s representativeness is negative (−37%), although statistically little significant. The regression line fitted to the representativeness curve confirms the same: the less successful parties tend to be more representative than the winning parties, although this dependence is rather irregular ($R^2 = 0.12$) and statistically little significant ($P_F = 0.0774$).

[Friendly 2002, p. 318] notes that ‘the task of detecting patterns of relations, trends, and anomalies is made considerably easier when “similar” variables are arranged contiguously’ — in our case, when neighboring parties have close policy profiles. The given party ordering is characterized by Figure 2, displaying the triangle of correlations between the party profiles. It is colored as a geographical map with brown mountains, green valleys and blue ocean depth (‘relief table’ [Tangian 2011, p. 107 ff.]). It plainly appears that close profiles of neighboring parties would imply a brown ridge of correlation peaks along the diagonal, which is not observed here. Therefore, our goal is to find a new party ordering with highly correlated profiles of neighboring parties. This ordering will also characterize the German actual political spectrum.

Table 1: German parties in the 2013 Bundestag election














Party logo	Party description	Number of Votes received	
		Number	%
	Union of Germany's two main conservative parties, Christlich Demokratische Union Deutschlands (Christian Democratic Union of Germany) founded in 1950 and Christlich-Soziale Union in Bayern (Christian Social Union of Bavaria) founded in 1945	635000	18157256 41.550
	Sozial-demokratische Partei Deutschlands (Social Democratic Party) founded in 1863	477000	11247283 25.737
	Die LINKE (The Left) founded in 2007 as the merger of East German communists and the Electoral Alternative for Labour and Social Justice (WASG), a left-wing breakaway from the SPD	64000	3752577 8.587
	BÜNDNIS 90/DIE GRÜNEN (Alliance 90/The Greens) founded in 1993 as the merger of DIE GRÜNEN (West Germany) and BÜNDNIS 90 (East Germany), both with a social-democratic background	60800	3690314 8.445
	Freie Demokratische Partei (Free Democratic Party) founded in 1948, liberal political party close to employers' organizations	60000	2082305 4.765
	Alternative für Deutschland (Alternative for Germany) founded in 2013, a conservative, euro-currency-sceptic party	14000	2052372 4.696
	Piratenpartei Deutschland (Pirate Party of Germany) founded in 2006, a part of international Pirate movement promoting the information society with a free access to all digital medias	31700	958507 2.193
	National-demokratische Partei Deutschlands (National Democratic Party of Germany) founded in 1964, a far-right German nationalist party	5000	560660 1.283
	FREIE WÄHLER (Free Voters) founded in 2009, a party of opposition to the EU financial policy	6000	422857 0.968
	Mensch Umwelt Tierschutz (Human Environment Animal Welfare) founded in 1993, a party promoting the introduction of animal rights into the German constitution	1000	140251 0.321
	Ökologisch-Demokratische Partei (Ecological Democratic Party) founded in 1982, an conservative environmentalist party	5700	127085 0.291
	Die Republikaner (The Republicans) founded in 1983, a national conservative party opposing to immigration	5800	91660 0.210
	Partei für Arbeit, Rechtsstaat, Tierschutz, Elitförderung und basisdemokratische Initiative (Party for Work, Rule-of-Law, Protection of Animals, Advancement of Elites, and Grassroot-Democratic Initiative) founded in 2004, a populist paradoxical party with totalitarian trends	10000	78357 0.179

Table 1: German parties in the 2013 Bundestag election (continued)

Party logo	Party description	Number of Votes received members		
		Number	%	
	Bürger-bewegung pro Deutschland (Pro Germany Citizens' Movement) founded in 2005, a far-right populist party opposing to illegal immigration and multi-national corporations and financial institutions	730	74311	0.170
	Bayernpartei (Bavaria Party) founded in 1946, a separatist Bavarian party advocating Bavarian independence within the European Union	500	57285	0.131
	Volks-abstimmung (Referendum party) founded in 1997, a party promoting direct democracy of Swiss type	1000	28667	0.066
	Marxistisch-Leninistische Partei Deutschlands (Marxist-Leninist Party of Germany) founded in 1982, an anti-revisionist party, referring to Marx, Engels, Lenin, Stalin and Mao Zedong	2300	25336	0.058
	RENTNER Partei Deutschland (German Party of Pensioners) founded in 2002, a party of social welfare state bridging interests of generations	750	25190	0.058
	Partei der Vernunft (Party of Reason) founded in 2009, a liberal party promoting the ideas of Austrian School of economics - minimal state, free market, decentralization of political power and subsidiarity	1000	25027	0.057
	Partei Bibeltreuer Christen (Party of Bible-abiding Christians) founded in 1989, a conservative evangelical party, opposing antisemitism, same-sex marriage and abortion	2700	18529	0.042
	Bündnis für Innovation und Gerechtigkeit (Alliance for Innovation and Justice) founded in 2010, a party of muslims promoting their integration	1000	17965	0.041
	Bürgerrechts-bewegung Solidarität (Civil Rights Movement Solidarity) founded in 1992, a part of the worldwide LaRouche (U.S. politician) Youth movement with republican orientation but promoting worldwide solidarity, e.g. abolishing debts of the Third World	1200	13131	0.030
	DIE FRAUEN (The Women) a feminist party founded in 1995 promoting rights of women	300	12522	0.029
	Partei der Nichtwähler (Party of Non-Voters) founded in 1998, a party with a social democratic background promoting improving representative democracy by introducing elements of direct democracy	400	11349	0.026
	Bündnis 21 / Rentnerinnen- und Rentner-Partei (Alliance 21 / Female and Male Pensioner Party) founded in 2007, promoting improving the pension, health and education systems	1050	8851	0.020
	Die Violettten — für spirituelle Politik (The Violet — for spiritual Policy) founded in 2001 claiming to represent 'alternative spiritual politics in the new age'	700	8248	0.019
	Familien-Partei Deutschlands (The Family Party of Germany) founded in 1983, a party promoting family values	600	7451	0.017
	Partei für Soziale Gleichheit, Sektion der Vierten Internationale (Party of Social Justice, Section of the Fourth International) founded in 1997, a Trotskyist party	300	4840	0.011

Table 2: Public opinion and party positions on topical policy questions

Questions	Question weights				Public opinion	
	Un- weight- ed	Google hits in K \log_2	1st expert Unger 0-3	2nd expert Graef 0-3	Pro- tago- nists %	An- tago- nists %
	1 Introduce a nationwide minimum wage	1	10.98	3	3	86
2 Parents of children who do not attend state-sponsored day care should receive a childcare subsidy	1	9.98	2	3	20	77
3 Introduce a general speed limit on highways	1	8.70	2	1	53	45
4 Germany should retain the Euro as its currency	1	9.28	2	3	69	27
5 Electricity prices should be more heavily regulated by the state	1	9.40	2	3	90	10
6 Video surveillance in public spaces should be expanded	1	6.58	3	2	81	18
7 Germany should introduce an unconditional basic income	1	9.48	3	1	80	20
8 Only organic agriculture should receive financial incentives	1	9.10	1	0	76	23
9 All children, regardless of cultural heritage, should receive equal education	1	5.42	1	3	33	60
10 The top income tax rate should be increased	1	8.27	2	3	75	22
11 Germany should leave NATO	1	6.98	1	2	52	36
12 No new construction of coal-fired energy plants	1	7.48	1	2	92	8
13 The ‘morning after’ pill must be available on prescription only	1	6.45	2	0	68	32
14 All banks in Germany should be nationalized	1	7.34	2	2	60	31
15 Germany should accept more refugees	1	8.75	3	2	39	56
16 Employees should be compensated by the state for the time spent for incapacitated relatives	1	6.58	3	2	?	?
17 Political parties that are unconstitutional should remain illegal	1	5.54	2	2	73	22
18 The level of federal student financial aid should be independent of the parents’ income	1	11.01	1	2	51	21
19 Border control should be re-introduced	1	8.63	1	1	48	52
20 A legal female quota should be introduced for companies’ board members	1	10.25	2	3	31	65
21 Financially stronger federal states should less support weaker ones	1	8.17	2	2	9	86
22 The legally mandated retirement age should be lowered again	1	11.77	3	3	73	17
23 The government should employ more people with immigrant background	1	7.77	2	1	?	?
24 Exports of munitions should be forbidden	1	7.71	3	1	78	20
25 Retain the tax law that favors spouses	1	8.39	2	1	81	16
26 Germany should champion Turkey’s bid for EU membership	1	8.79	1	1	27	68
27 Bundestag members should reveal their exact auxiliary income	1	5.95	1	1	76	20
28 Energy-intensive industries should bear more of the costs of the transition to renewable energy	1	8.87	1	3	81	15
29 Recipients of long-term unemployment benefits should receive less if they turn down a job offer	1	7.65	2	3	50	50
30 The state should continue to collect tithes on behalf of religious institutions	1	9.81	0	0	31	69
31 All citizens should be required to enroll in the public health insurance system	1	14.95	3	3	83	16
32 Every state in the Euro zone should be liable to pay its own debts	1	10.50	1	3	52	38
33 Homosexual couples should be allowed to adopt	1	5.75	1	1	63	30
34 Abolish the collection of communication data (e.g. telephone, internet) without probable cause	1	9.61	3	2	65	30
35 By new lettings, the rental price increase should be limited	1	6.75	2	3	73	25
36 German citizens should be allowed to have additional nationalities	1	7.29	1	3	42	53
37 Institute a passenger-car toll on the national highways	1	9.95	1	1	22	57
38 Introduce referenda at the federal level	1	8.91	1	1	87	11
Representativeness index, %						

Table 2: Public opinion and party positions on topical policy questions (continued)

Questions	Party positions																												
	CDU/CSU	SPD	DIE LINKE	GRÜNE	FDP	AFD	PIRATEN	NPD	FREIE WÄHLER Tierschutzpartei	ÖDP	REP	Die PARTEI pro Deutschland	BP	Volksabstimmung	MLPD	RENTNER	Partei der Vernunft	PBC	BIG	BüSo	DIE FRAUEN Nichtwähler	Bündnis 21/RRP	DIE VIOLETTEN	FAMILIE	PSG				
1	0	1	1	1	0	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	1	1	?	1	1	1	1		
2	1	0	0	0	?	?	0	0	1	0	0	1	1	0	?	1	1	0	0	0	1	0	1	0	0	0	1	1	0
3	0	0	1	1	0	0	?	0	0	1	1	0	0	0	?	1	0	0	0	0	0	1	?	0	1	1	1	1	
4	1	1	1	1	1	0	1	0	1	1	1	0	1	0	0	1	?	1	0	1	1	0	?	1	1	1	1	0	
5	0	1	1	?	0	0	?	1	1	1	0	0	1	1	1	?	1	0	1	1	0	1	1	1	1	1	1	1	
6	1	?	0	0	0	0	?	1	0	0	1	0	1	0	1	0	1	0	1	0	1	0	0	?	?	1	0	1	0
7	0	0	?	?	0	0	1	0	0	1	0	0	1	0	0	?	0	0	0	1	0	?	?	0	1	0	1	0	
8	0	?	?	?	0	0	1	1	0	1	1	0	1	1	0	1	0	0	0	0	0	0	1	?	1	1	1	0	
9	1	1	1	1	1	1	1	0	1	1	1	0	1	0	1	?	1	1	0	?	1	1	1	?	1	1	1	1	
10	0	1	1	1	0	0	?	1	0	1	1	0	1	0	0	?	1	1	0	0	1	0	1	0	1	1	1	1	
11	0	0	1	0	0	0	0	1	0	0	0	?	0	1	0	?	1	0	1	0	0	1	1	0	0	?	0	1	
12	0	0	1	1	0	?	1	1	0	1	1	0	?	0	1	?	1	0	0	?	1	0	1	?	1	1	1	?	
13	1	0	0	0	?	1	0	1	1	0	1	1	0	1	1	1	0	0	0	1	1	1	0	?	1	1	1	0	
14	0	0	?	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1	
15	0	1	1	1	?	0	1	0	0	?	1	0	1	0	0	?	1	?	0	?	1	1	1	?	?	1	1	1	
16	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	?	1	1	1	1	
17	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	?	1	0	1	1	1	?	1	1	1	1	0	
18	0	?	1	1	1	0	1	1	1	0	1	0	1	1	1	0	0	0	0	1	0	0	1	?	0	1	1	1	
19	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	?	0	0	0	0	?	0	
20	?	1	1	1	0	0	?	0	0	1	0	0	1	0	0	0	?	0	0	0	?	?	1	?	0	1	1	0	
21	?	0	0	0	0	1	?	?	?	0	?	1	0	?	1	?	0	0	1	?	0	0	0	?	0	0	1	0	
22	0	1	1	0	?	0	?	1	1	1	0	0	1	1	1	1	1	1	0	0	1	1	?	?	1	1	?	1	
23	1	1	1	1	1	?	1	0	1	1	1	0	1	0	0	?	1	?	0	0	1	1	1	?	?	1	1	1	
24	0	0	1	?	0	0	1	1	0	1	1	?	1	1	?	?	1	0	0	?	0	?	1	1	0	1	1	1	
25	1	?	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	?	1	0	?	0		
26	0	1	1	1	?	0	?	0	0	0	?	0	0	0	0	1	0	0	0	0	1	0	?	?	?	1	0	0	
27	?	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	
28	0	?	1	1	0	0	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	
29	1	1	0	0	1	1	0	?	1	?	0	1	0	0	1	0	0	1	1	1	0	0	0	?	1	0	0	0	
30	1	1	0	?	?	1	0	?	1	0	1	0	0	1	1	?	0	0	0	1	1	?	0	?	0	0	0	0	
31	0	1	1	1	0	0	1	1	0	0	1	0	?	1	0	1	1	1	0	0	1	0	1	?	1	0	1	1	
32	1	?	0	0	1	1	0	1	1	1	1	1	?	1	1	1	0	1	1	1	0	1	0	?	1	1	1	0	
33	0	1	1	1	1	0	1	0	?	1	?	0	1	0	0	1	1	1	1	0	0	0	1	1	1	1	?	1	
34	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	
35	1	1	1	1	0	0	1	1	1	1	1	0	1	0	1	1	1	1	0	0	1	1	1	?	1	1	1	1	
36	1	0	0	0	0	1	0	1	0	1	?	1	0	1	1	?	0	1	0	1	0	?	0	0	0	0	0	0	
37	?	0	0	?	0	?	0	0	0	0	0	1	1	0	1	0	0	?	0	0	0	0	?	?	0	0	0	0	
38	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	

Representativeness index, % 36 59 70 61 44 39 68 70 56 68 63 40 71 65 49 73 69 65 39 52 62 47 72 73 74 66 68 69

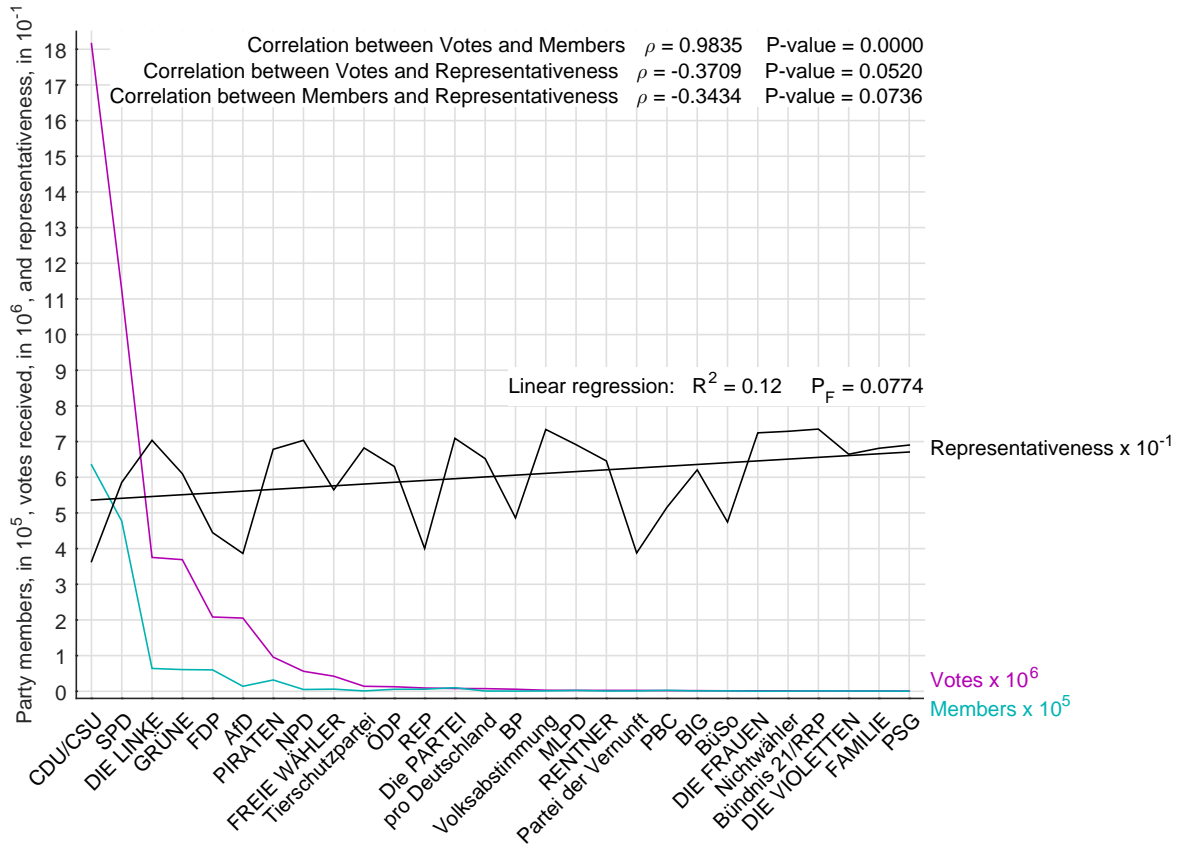


Figure 1: Party sizes, votes received, and representativeness of German parties in 2013

3 Principal Component Analysis Solution

Following [Friendly 2002, Friendly and Kwan 2003], we find a contiguous party ordering by means of Principal Component Analysis of the correlation matrix (the correlation triangle in Figure 2 is its bottom-left half). The upper plot in Figure 3 shows the location of the party vectors in the two-dimensional space of the first two components that cover 83.4% of the total variance. The correlation between the party profiles is approximated by the cosine of the angle between the party vectors. Thereby we obtain a circular ordering, with neighboring parties having correlated policy profiles. Cutting this circular ordering at the greatest angle (between the vectors of the far-left Trotskyist party PSG — and far-right nationalist party NDP) and going clockwise, we obtain a plausible left-right party ordering.

The correlation triangle with the new party ordering is shown in Figure 4. It has the desired ridge of brown correlation peaks of neighboring parties along the diagonal, green low correlation ‘valleys’ of more distant parties, then a blue negatively correlated band of the parties opposite in the circular ordering, and, finally, the green bottom-left vertex, indicating that the far-left and far-right parties have something in common.

The figure $S = -3592$ beyond the correlation triangle, the total weighted squared distance of the cells to the diagonal, characterizes the ordering contiguity. For each cell, its distance to the diagonal is the minimal number of cells to the diagonal, that is, the distance of the (i, j) -

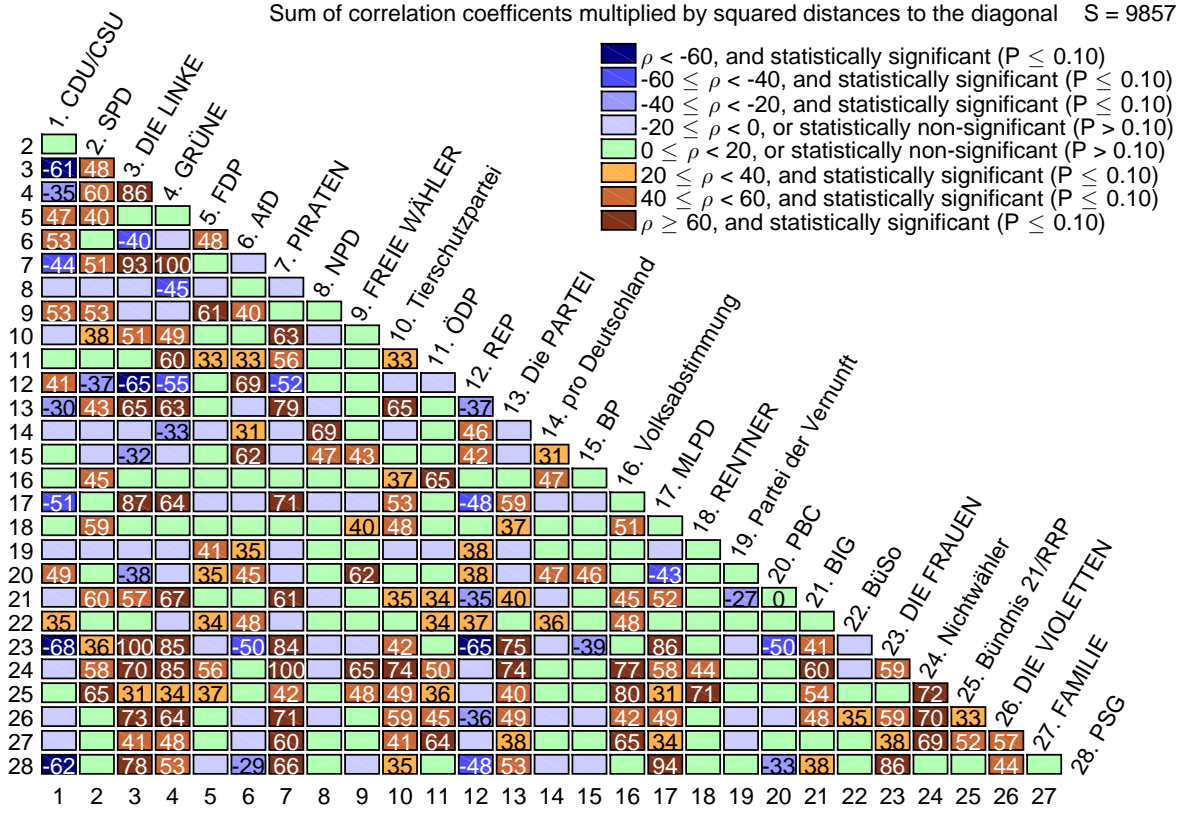


Figure 2: Triangle of party profile correlations (in %), for the parties ordered by votes received

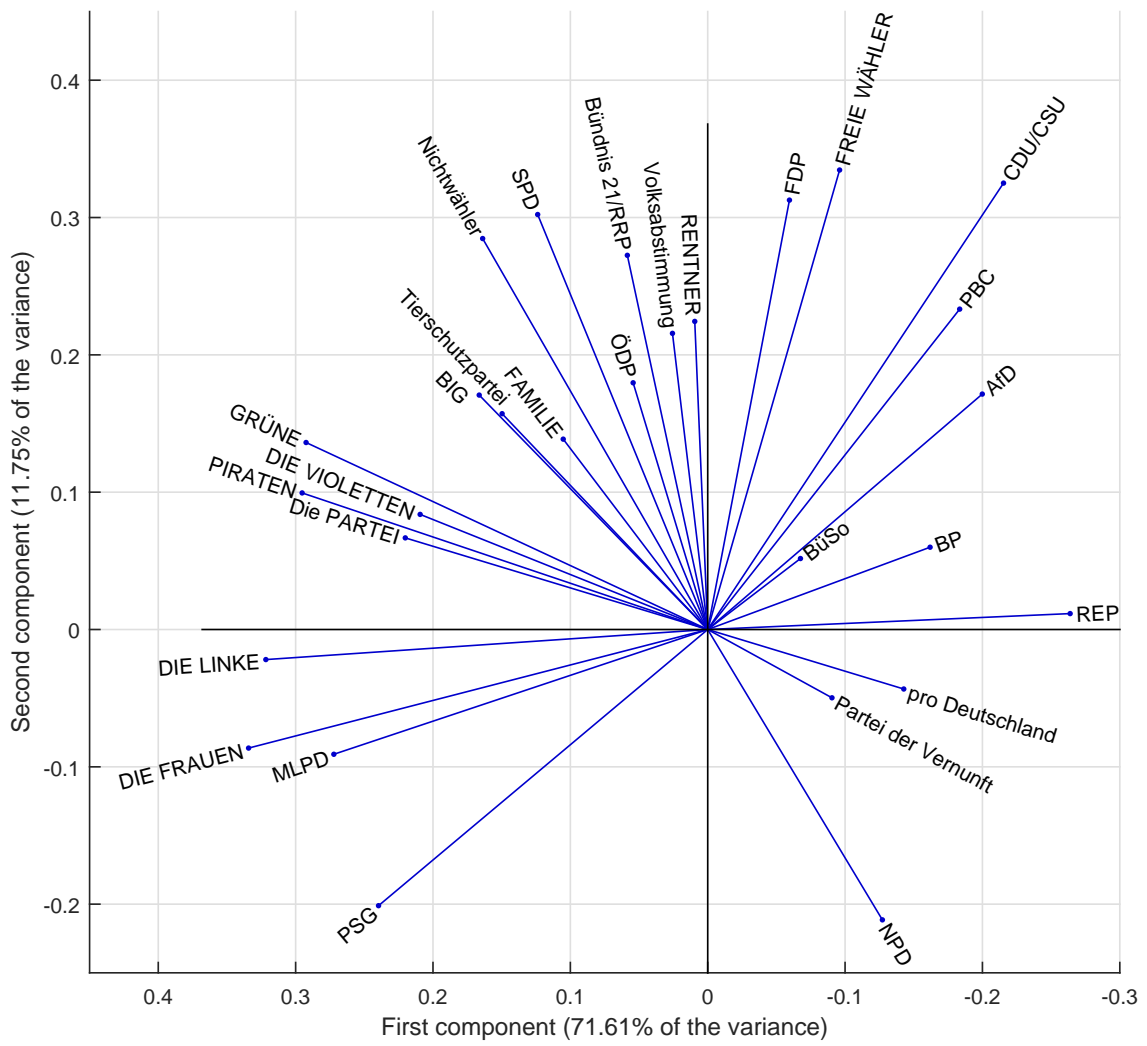
cell, $i > j$, to the diagonal is $i - j - 1$ (we refer to the so-called Manhattan distance). The squared distances are weighted with the corresponding correlation coefficients ρ_{ij} , so that the total weighted distance of cells to the diagonal is

$$S = \sum_{i>j} \rho_{ij} \times 100\% \times (i - j - 1)^2 .$$

The much better contiguity of the party ordering in Figure 4 is reflected by $S = -3592$, compared with $S = 9857$ in Figure 2.

The two plots below the correlation triangle in Figure 4 depict the representativeness curve for two versions of the party scale. In the first plot, the distances between the parties' ticks are made proportional to the angle between the party vectors in Figure 3, i.e. the closer the ticks, the closer the party profiles. The party scale in the bottom plot is uniform, that is, the closeness of the party profiles is not taken into account. In both plots of Figure 4, the representativeness curve exhibits visible trends. Indeed, the regression lines fitted to the representativeness curve have much superior fitting parameters R^2 and P_F than in Figure 2. The statistically significant descent of the regression line in both plots ($P_F < 0.01$) indicates at a higher representativeness of left parties and lower representativeness of right parties.

To reveal a trend for the circular party ordering, we consider a special circular regression model with the same fitting parameters as for the linear regression used so far (to make both models comparable). For this purpose, we locate the vectors of the independent variable (party



$$Z = 59.93 + 12.15 * X - 2.64 * Y \quad R^2 = 0.54 \quad P_F = 0.0001$$

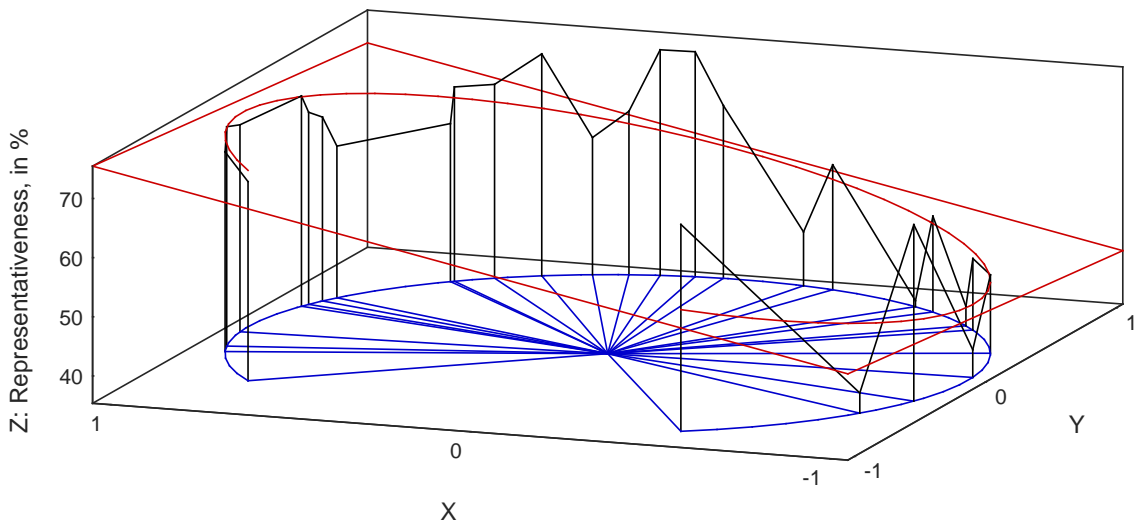


Figure 3: Principal component analysis solution. (1) Eigenvector plot for the correlation matrix of the party profiles; to visualize the left-right orientation, the direction of the first axis is reversed. Distances between party profiles are proportional to the angles between the party vectors. (2) Circular regression model to fit a sinusoidal to the party representativeness curve. The circumference is broken at the link with the largest distance between party profiles.

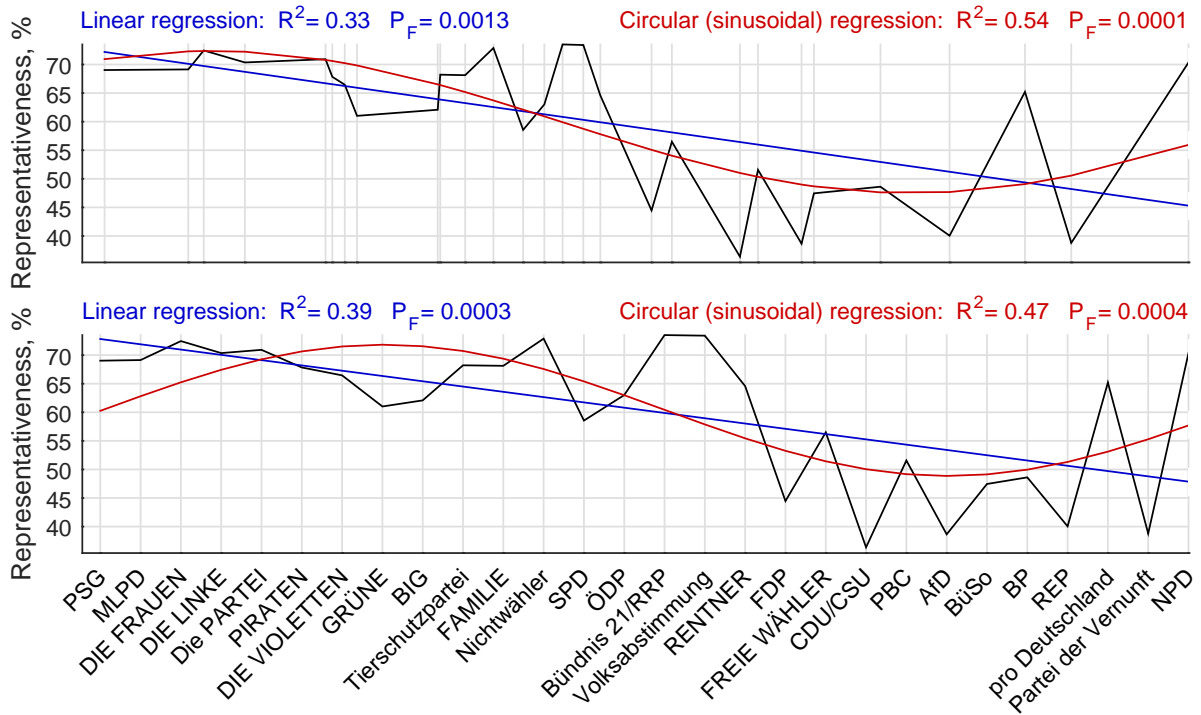
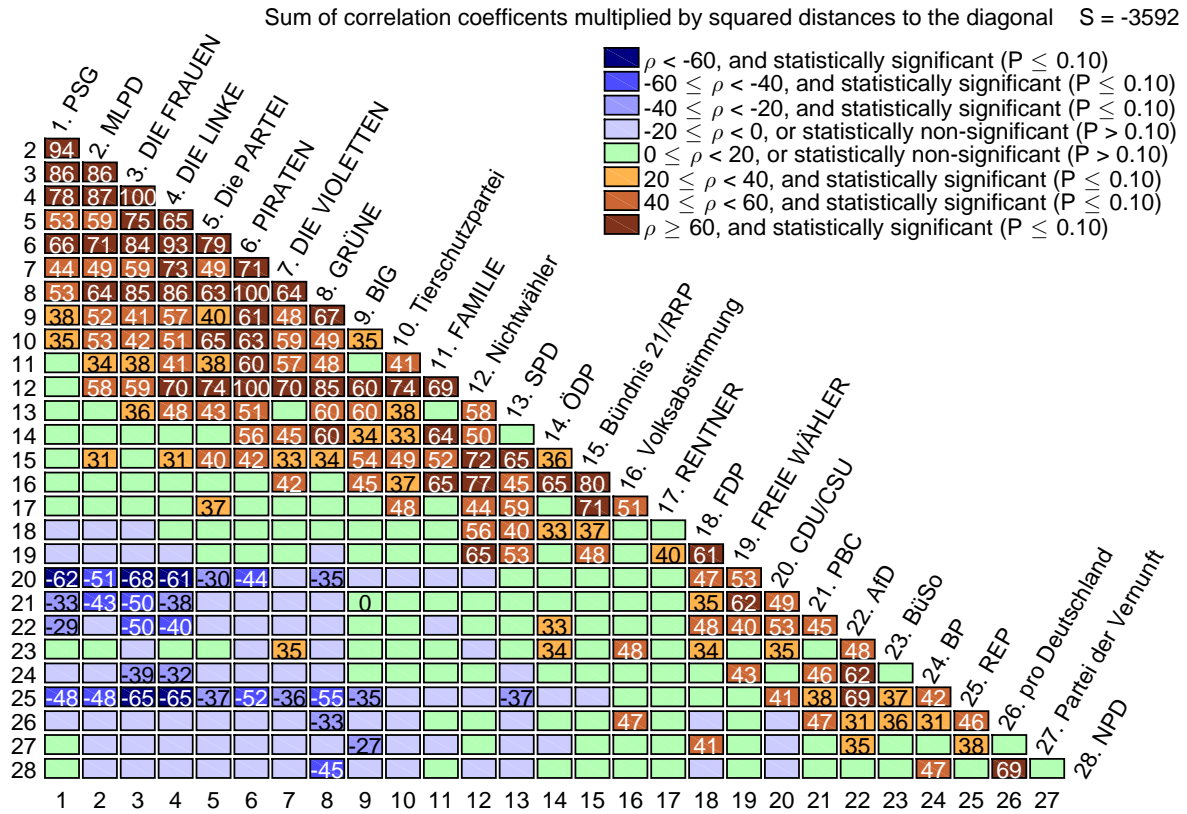


Figure 4: Principal component analysis solution. (1) Triangle of party profile correlations (in %), for the party ordering found. (2) The representativeness curve of the parties and its regression for this party ordering with taking into account the distance between neighboring parties (visualized by vertical grid lines with variable distances). (3) The representativeness curve of the parties and its regression for this party ordering without taking into account the distance between neighboring parties (visualized by vertical grid lines with equal distances).

vectors) on a circumference in a two-dimensional XY -plane, and the dependent variable (party representativeness index) locate in the third Z -dimension exactly above the party vectors; see the bottom plot in Figure 3. Then we fit a regression plane to the resulting three dimensional vectors in the XYZ -space, and obtain the predicted values at the intersection of the regression plane with the cylinder over the given circumference. This intersection curve transferred to the plots of Figure 4 is the regression sinusoid. The quality of fit of the regression sinusoid is even better than that of the regression line, reflecting the circular nature of the party ordering. This regression model introduces some corrections to our previous statement about the superior representativeness of left parties. It looks that the most representative are the moderate left parties, then come far-left and far-right parties, and the least representative are centrist and moderate conservative parties.

4 Traveling Salesman Problem Solution

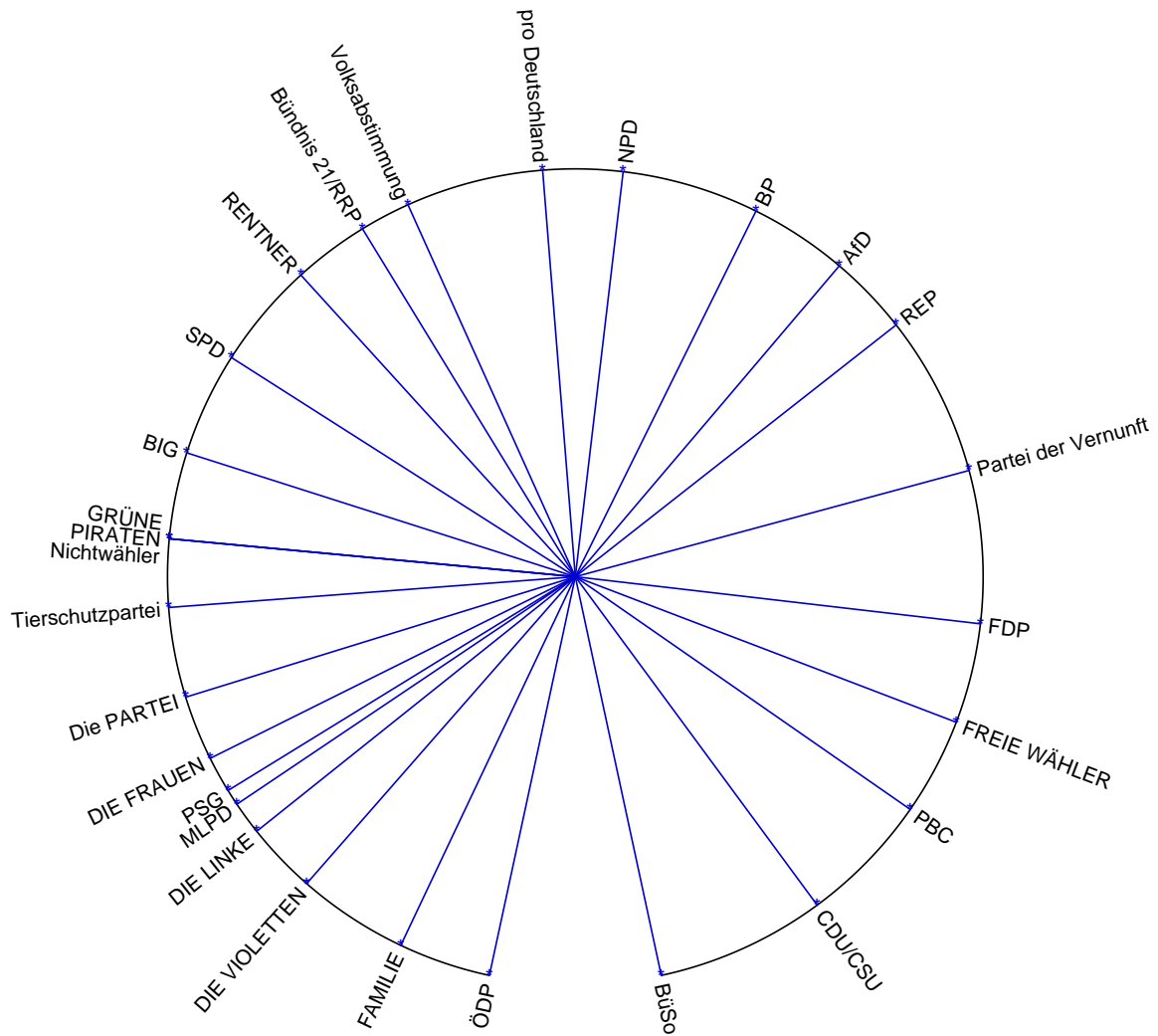
The PCA method performs dimensionality reduction: a 38-dimensional space with 28 policy profile vectors of 28 parties is quite accurately approximated with a two-dimensional space, covering over 83% of the total variance. This model reveals that the German political spectrum can be approximately regarded as the left-right ideological axis rolled in a circumference. Let us see, which contiguous circular axis can be obtained directly, not dealing with dimensionality reduction.

For this purpose we reformulate our task as a traveling salesman problem. We have to find the shortest cyclic itinerary through 28 destinations, that are in our case 28 parties, visiting each only once. As the distance table, we use our correlation triangle somewhat modified. The distances between parties with highly correlated profiles should be close to 0, and between parties with negatively correlated profiles relatively large. Therefore, we derive the distance d_{ij} between parties i, j from the correlation coefficient ρ_{ij} with the opposite sign as follows

$$d_{ij} = 1 - \rho_{ij} .$$

The upper plot in Figure 5 shows the shortest circular itinerary through the 28 parties. This way we obtain both linear and circular ordering of contiguous parties. The longest arc is removed to show the shortest itinerary through the 28 parties without returning to the starting point. The lower plot in Figure 5 illustrates the construction of the regression sinusoid for the new circular ordering, which follows the same principles as described in the previous section.

Figure 6 visualizes the properties of the new linear and cyclic party orderings. The quality of fit of regression lines and sinusoids to the representativeness curve in both bottom plots is superior to that in Figure 4, because the party ordering is optimized with respect to contiguity only, being no longer subordinated to dimensionality reduction. However, this has its drawbacks: the correlation triangle is not that structurally layered as in Figure 4, and the overall concentration of correlation peaks along the diagonal is weaker, having $S = 865$ compared with $S = -3592$ in Figure 4.



$$Z = 57.18 - 13.48 * X + 1.00 * Y \quad R^2 = 0.66 \quad P_F = 0.0000$$

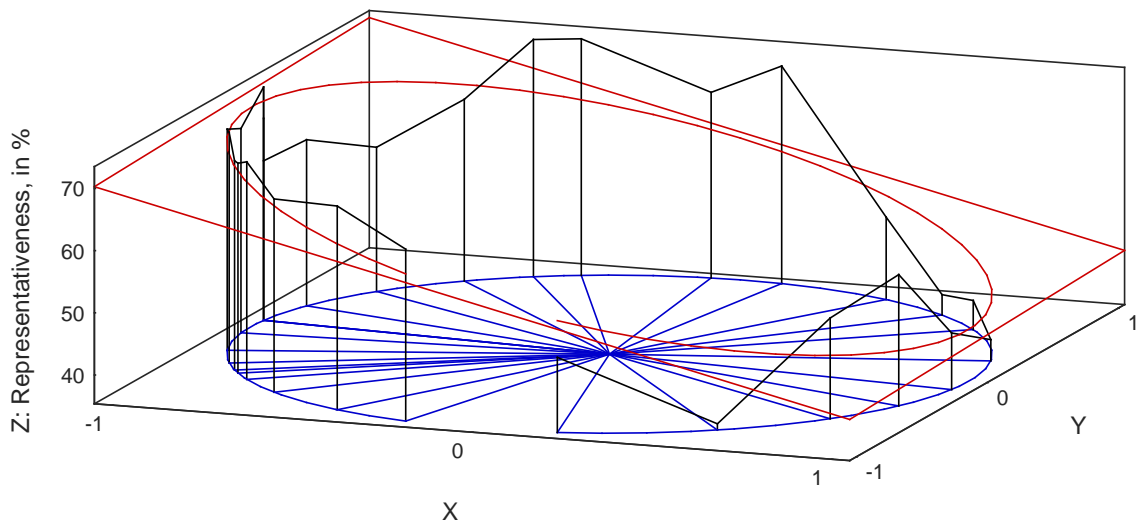


Figure 5: Traveling salesman problem solution. (1) The parties are ordered to minimize the total distance between neighboring party profiles. The distances are proportional to arc lengths (angles). The circumference is broken at the link with the largest distance between party profiles. (2) Circular regression model to fit a sinusoidal to the party representativeness curve

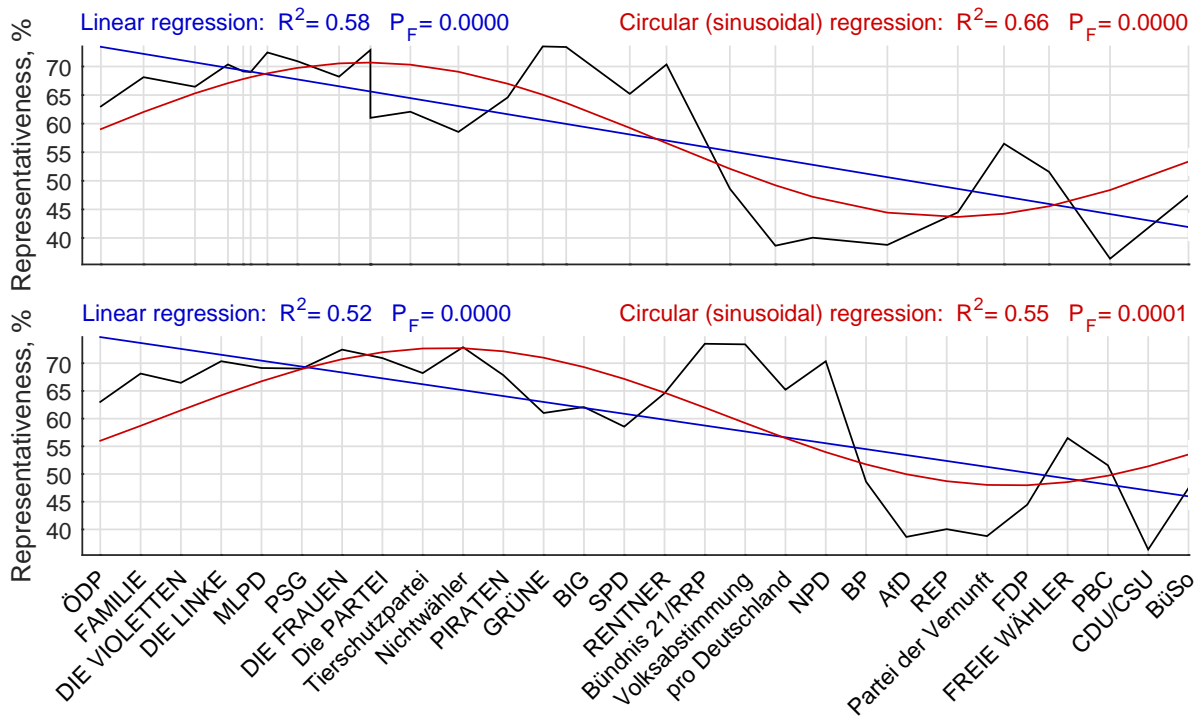
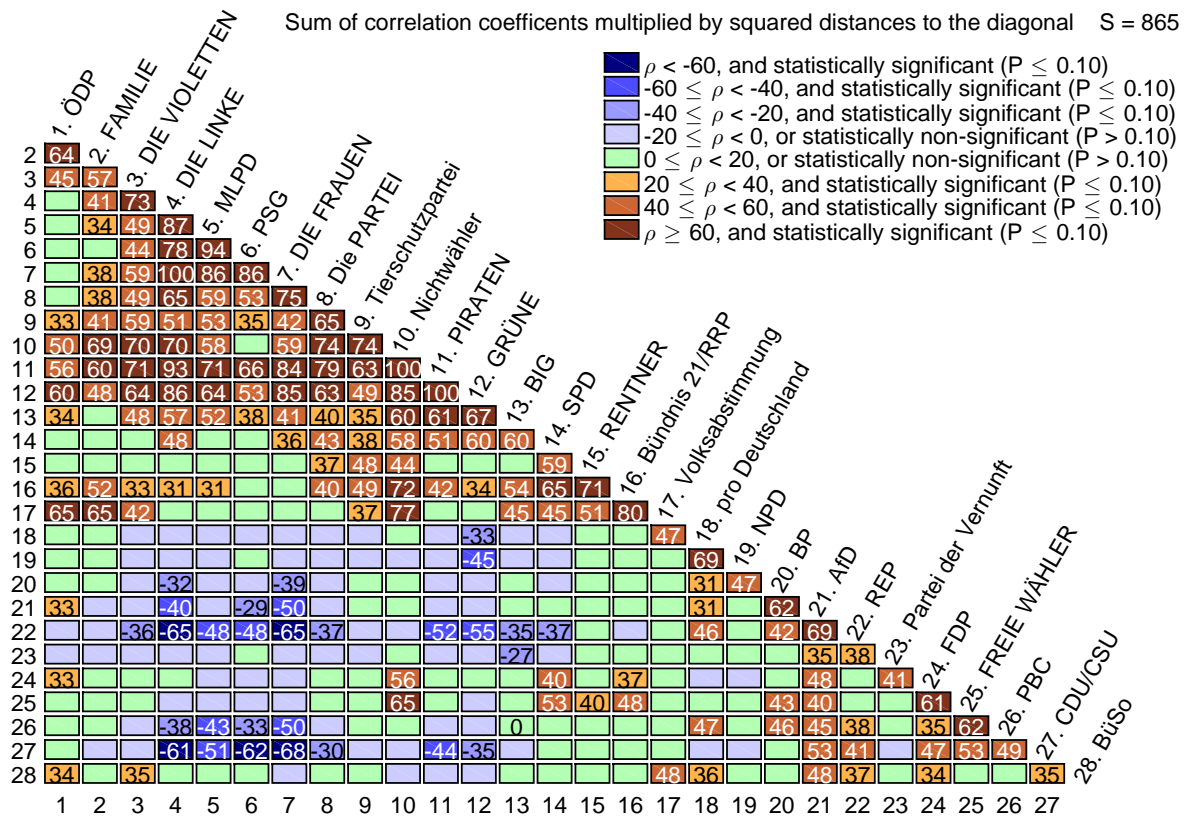


Figure 6: Traveling salesman problem method. (1) Triangle of party profile correlations (in %), for the party ordering found. (2) The representativeness curve of the parties and its regression for this party ordering with taking into account the distance between neighboring parties (visualized by vertical grid lines with variable distances). (3) The representativeness curve of the parties and its regression for this party ordering without taking into account the distance between neighboring parties (visualized by vertical grid lines with equal distances).

5 Weighted least squares solution

Now we optimize the party ordering with regard to the concentration of correlation peaks along the diagonal of the correlation triangle. For this purpose we minimize the weighted squared distance of cells of the correlation triangle to its diagonal, i.e., we minimize the following expression with which we characterize the correlation triangle

$$S = \sum_{i>j} \rho_{ij}(i - j - 1)^2 \rightarrow \min$$

The optimization is made iteratively as long as no further progress is attained. At each step, a loop on 28 parties is performed. In each loop, the current party is relocated in the ordering to minimize the sum S (this is also implemented as a loop on 28 alternative positions).

The new party ordering and its properties are visualized in Figure 7. As one can see, $S = -5376$ is the least compared with that in Figures 2, 4 and 6. However, the quality of fit of regression lines and sinusoids is inferior to that in Figures 4 and 6 (the distances between the parties in the middle plot of Figure 6 are defined as in Section ‘Traveling salesman problem solution’). This can be interpreted that the German political spectrum cannot be approximated by a single linear axis with missing circularity (which requires a two-dimensional room).

6 Weighted largest squares solution

The task we formulate now is similar to that from the previous section, but we change the criterion of optimization. Instead of minimizing the total distance of correlation peaks to the correlation triangle diagonal, we maximize the distance of correlation peaks from the bottom-left vertex of the correlation triangle. In other words, we perform the same procedure as previously but with maximizing the following expression (recall that if n is the number of parties then $n - 1$ is the distance of the bottom-left vertex of the correlation triangle to its diagonal):

$$S_{\max} = \sum_{i>j} \rho_{ij}[n - 1 - (i - j)]^2 = \sum_{i>j} \rho_{ij}(27 - i + j)^2 \rightarrow \max$$

The new party ordering and its properties are visualized in Figure 8. As one can see, $S = -4409$ is not as small as in Figure 7, but smaller than in Figures 2, 4 and 6. The quality of fit of regression lines is a little worse than in Figure 7 but the regression sinusoids are fitted better to the representativeness curve. The bottom-left corner of the correlation triangle is not as filled with dark blue cells with most negative correlation coefficients as in Figure 7, meaning that a certain circularity in the party ordering is somehow revealed (the vertex cell binding the far-left and far-right ends is green!). Therefore, the sinusoids in the two bottom plots of Figure 8 are fitted to the representativeness curve better than in Figure 7. The ‘revival’ of circularity in the party ordering makes it quite similar to the ordering in Figure 4 obtained by means of dimensionality reduction.

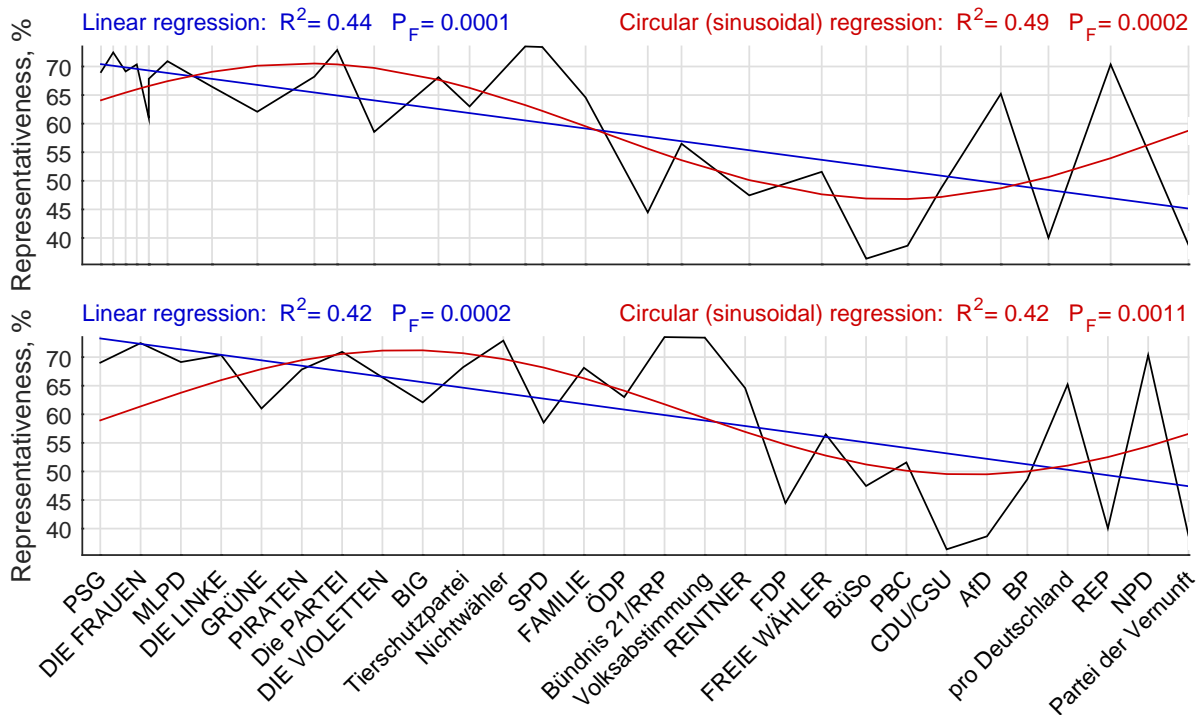
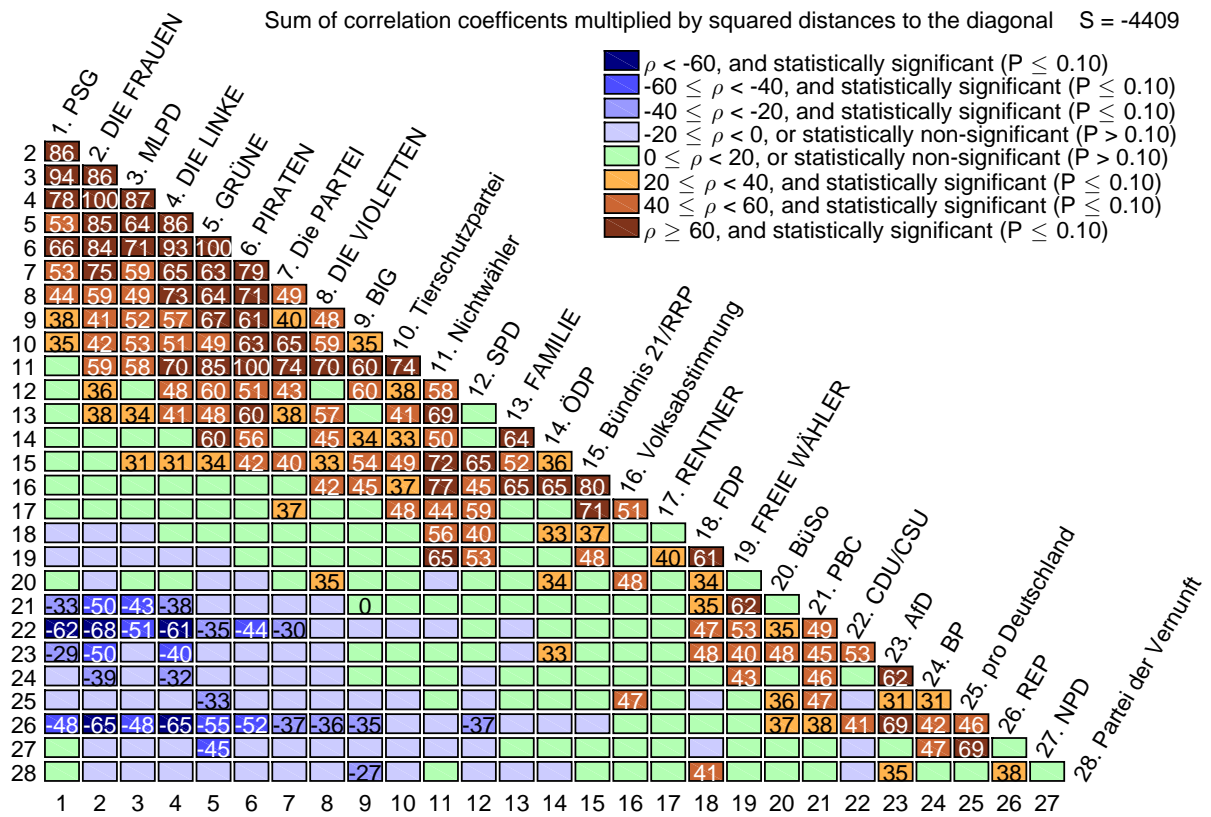


Figure 8: Weighted largest squares method. (1) Triangle of party profile correlations (in %), for the party ordering found. (2) The representativeness curve of the parties and its regression for this party ordering with taking into account the distance between neighboring parties (visualized by vertical grid lines with variable distances). (3) The representativeness curve of the parties and its regression for this party ordering without taking into account the distance between neighboring parties (visualized by vertical grid lines with equal distances).

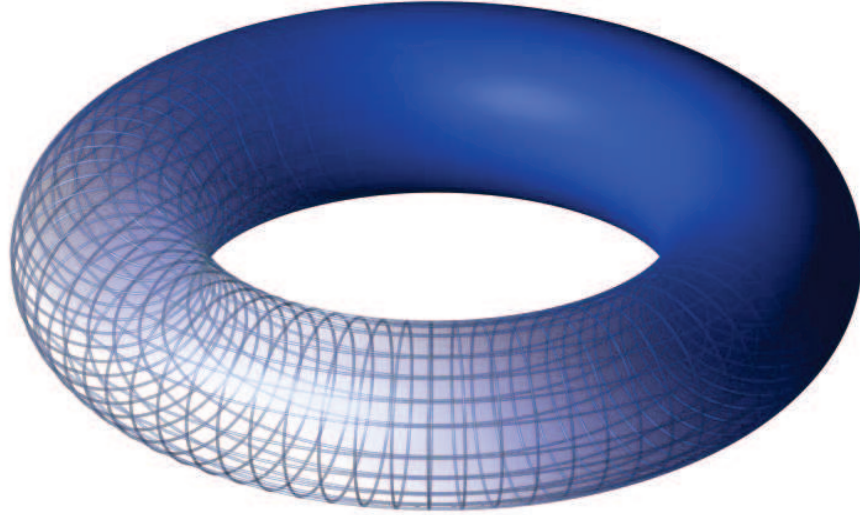


Figure 9: Torus (a bagel-like body) whose form illustrates the spatial location of party policy vectors along the circular axis with minor deviations

7 Conclusions

The ‘objective’ ordering of 28 German parties, obtained purely formally without any normative assumption, brings us to the known left–right ideological axis rolled in a circumference, making the extreme left-hand and right-hand ends meet. In the policy space, party profile vectors may have minor deviations from this rolled axis, making a bagel-shaped ‘cloud of observations’ (this type of body is known in geometry as torus). Due to the deviations, the circular axis gets volume, turning into a circular tube, as shown in Figure 9 for one-dimensional deviations. When the deviations are multi-dimensional, as in our study, the principle remains the same but the tube should be imagined in a multi-dimensional space.

The most plausible and accurate left–right axis is obtained by dimensionality reduction of the policy space with the Principal Component Analysis, as compared with three other methods considered. The consequent party ordering exhibits a statistically highly significant dependence between the party’s ideological platform and its representativeness, with the left parties being more representative than the right ones. The even more accurate circular representation of the German political spectrum demonstrates that the extreme left parties tend to be less representative than moderate left parties, and the far-right parties tend to be more representative than moderate right (conservative) parties.

Basing on this empirical evidence, we conclude that the left-right characterization of parties which reflects the class opposition remains valid, being in no case outdated. Our study also indicates at the raising far-right wing, whose representativeness already surpasses that of the conservative party that has won the 2013 election. As for the moderate left parties, it looks that their superior representativeness can help to restore their influence, which sharply declined after the collapse of communism in the end of the 20th century.

8 References

- Blattberg Ch (2009) Political philosophies and political ideologies. In: Blattberg Ch. *Patriotic Elaborations: Essays in Practical Philosophy*. McGill–Queen’s University Press, Montreal
- Bobbio N, Cameron A (1996) *Left and right: the significance of a political distinction*. Chicago, University of Chicago Press
- Budge I, Klingemann HD, Volkens A, Bara J, Tanenbaum E (2001) *Mapping policy preferences: estimates for parties, electors and governments 1945–1998*. Oxford University Press, Oxford
- Budge I, McDonald MD (2007) Election and party system effects on policy representation: bringing time into a comparative perspective. *Electoral Studies*, 26(1), 168–179
- Bundeswahlleiter (2013) Ergebnisse der Wahl zum 18. Deutschen Bundestag. <http://www.bundeswahlleiter.de/de/bundestagswahlen/BTWBUND09/>
- Bundeszentrale für politische Bildung (2013). Wahl-O-Mat. <http://www.bpb.de/politik/wahlen/wahl-o-mat/>
- Downs A (1957) *An Economic Theory of Democracy*. Harper and Row, NY
- EU profiler (2009). <http://www.euprofiler.eu/>
- Friendly M (2002) Corrgrams: exploratory displays for correlation matrices. *American statistician*, 56(4), 316–324. <http://www.datavis.ca/papers/corrgram.pdf>
- Friendly M, Kwan E (2003) Effect ordering for data display. *Computational Statistics and Data Analysis*, 43, 509–539. <http://www.datavis.ca/papers/effect/effect.pdf>
- Garzia D, Marschall S (eds) (2014) *Matching Voters with Parties and Candidates: Voting Advice Applications in a Comparative Perspective*. ECPR Press, Colchester
- Gauchet M (1996) Right and left. In: Nora P, Kritzman LD (Eds.) *Realms of Memory: Rethinking the French Past. Vol 1 Conflicts and Divisions*. Columbia University Press, New York, 240–298
- Giddens A (1994) *Beyond Left and Right, the Future of Radical Politics*. Stanford University Press, Stanford CA
- Gill J, Hangartner D (2010) Circular data in political science and how to handle it. *Political Analysis*, 18(3), 316–336. <http://artsci.wustl.edu/~jgill/papers/mpq009.pdf>
- Kieskompas (2006). <http://www.kieskompas.nl/>
- Klingemann HD, Volkens A, Bara JL, Budge J, McDonald MD (eds) (2006) *Mapping Policy Preferences II. Estimates for Parties, Electors, and Governments in Eastern Europe, European Union, and OECD 1990–2003*. Oxford University Press, Oxford

- Knapp A, Wright V (2001) *The Government and Politics of France*. Routledge, New York
- Left–right politics (2015) Wikipedia. http://en.wikipedia.org/wiki/Left%E2%80%93right_politics
- Lijphart A (1999) *Patterns of Democracy*. Yale University Press, New Heaven–London
- Linhart E, Shikano S (2007) Die Generierung von Parteipositionen aus vorverschlussten Wahlprogrammen für die Bundesrepublik Deutschland, MZES working paper. Mannheim, Mannheimer Zentrum für Europäische Sozialforschung
- Lipset SM (1960) *Political Man: The Social Bases of Politics*. Doubleday, Garden City NY
- Mahoney J, Coogle CL, Banks PD (1984) Values in presidential inaugural addresses: A test of Rokeach’s two-factor theory of political ideology. *Psychological Reports*, 55 (3), 683–686
- Manin B (1997) *The Principles of Representative Government*. Cambridge University Press, Cambridge
- Marks G, Hooghe L, Nelson M, Edwards E (2006) Party competition and European integration in the East and the West. Different structure, same causality. *Comparative Political Studies* 39, 155–175.
- Marx K (1867) *Das Kapital*. Band I. Verlag von Otto Meisner, Hamburg. http://www.mlwerke.de/me/me23/me23_000.htm
- Matthews GVT (1974) On bird navigation, with some statistical undertones. *J Royal Stat Soc B*, 36, 349–64
- Mitchell BP (2007) *Eight Ways to Run the Country: A New and Revealing Look at Left and Right*. Praeger, Westport CN
- Political spectrum (2015) Wikipedia. http://en.wikipedia.org/wiki/Political_spectrum
- Pro demos: Hois voor democratie en rechtsstaat (2015) StemWijzer. <http://www.stemwijzer.nl/>
- Rabinowitz G, MacDonald SE (1989) A directional theory of issue voting. *Am Pol Sc Rev* 83(1), 93–121
- Rous GL, Lee DE (1978) Freedom and equality: two values of political orientation. *J Communication*, 28, 45–51
- Ruyters J (2005) *Canadian and World Politics*. Emond Montgomery Publications Limited, Toronto
- Schofield NJ (1985) *Social Choice and Democracy*. Springer, New York

- Schofield NJ (1993) Party competition in a spatial model of coalition formation. In: Barnett WA, Hinich MJ, Schofield NJ (eds) *Political Economy: Institutions, Competition, and Representation*. Cambridge University Press, Cambridge, 135–174
- Smithies A (1941) Optimum location in spatial competition. *J Polit Econ*, 49, 423–39
- Streeck W (1999) *Competitive Solidarity: Rethinking the 'European Social Model'*. MPIfG Working Paper 99/8, September 1999. Reprinted in: Leibfried S, Mau S (2008) (eds) *Welfare States: Construction, Deconstruction, Reconstruction. Vol II: Varieties and Transformations*. Edward Elgar, Cheltenham, 549–565
- Sulakshin S (2010) A quantitative political spectrum and forecasting of social evolution. *Int J Interdisciplinary Soc Sc*, 5(4), 55–66
- Tangian A (2011) *Flexicurity and Political Philosophy*. Nova, New York
- Tangian A (2013) 2013 Election to German Bundestag from the Viewpoint of Direct Democracy. WSI-Diskussionspapier 186. Hans-Böckler-Foundation, Düsseldorf. http://www.boeckler.de/pdf/p_wsi_disp_186.pdf
- Tangian A (2014) *Mathematical Theory of Democracy*. Springer, Berlin–Heidelberg
- Voda P (2014) Class voting in West and East. Paper at the 8th ECPR General Conference, University of Glasgow, September 3–6, 2014. <http://ecpr.eu/events/paperlist.aspx?EventID=4&SectionID=28&PanelID=288>
- Volkens A, Bara J, Budge I, McDonald MD, Klingemann HD (eds) (2013) *Mapping Policy Preferences from Texts: Statistical Solutions for Manifesto Analysts*. Oxford University Press, Oxford
- Vote Match Europe (2015). <http://www.votematch.eu/>
- Ware A (1996) *Political Parties and Party Systems*. Oxford University Press, Oxford
- Weber M (1921) *Economy and Society*, ed Roth G and Wittich C, 2 vols. University of California Press, Berkeley and Los Angeles, 1978. <http://s0ftpedia.net/files/max%20weber%20economy%20and%20society%20pdf&id=my>
- Wilson MS (2004) Values and political ideology: Rokeach's two-value model in a proportional representation environment. *New Zealand J Psychology*, 33(3), 155–162
- WZB (2015) *The Manifesto Project*. WZB, Berlin. <https://manifestoproject.wzb.eu/information/information>

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