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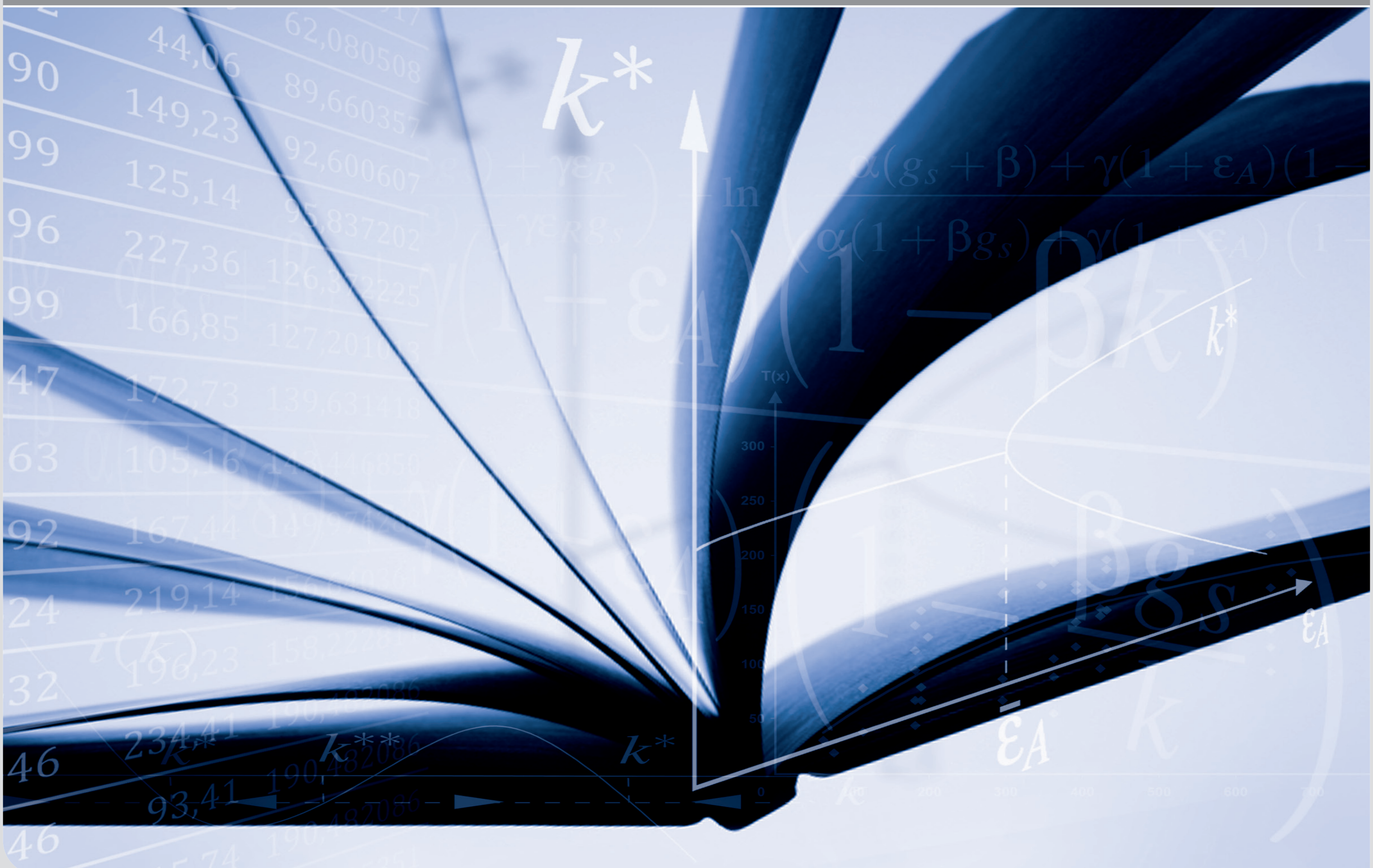

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Option Pricing with Regime Switching Tempered Stable Processes

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Abstract In this paper we will introduce a hybrid option pricing model that combines the classical tempered stable model and regime switching by a hidden Markov chain. This model allows the description of some stylized phenomena about asset return distributions that are well documented in financial markets such as time-varying volatility, skewness, and heavy tails. We will derive the option pricing formula under the this model by means of Fourier transform method. In order to demonstrate the superior accuracy and the capacity of capturing dynamics using the proposed model, we will empirically test the model using call option prices where the underlying is the S&P 500 Index.

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1 Introduction

The skewness and heavy tail property in the return distribution and the time varying volatility of return process are two major issues for derivative pricing in finance. Tempered stable processes have been used to capture the skewness and heavy tail property; see Boyarchenko and Levendorskiĭ (2000), Barndorff-Nielsen and Levendorskii (2001), and Carr *et al.* (2002). By the help of the fast Fourier transform method, the tempered stable processes have been successfully applied to the European option pricing; see Carr and Madan (1999) and Lewis (2001).

The time varying volatility can be modeled by the GARCH model by Engle (1982) and Bollerslev (1986) and the regime switching time series model by Hamilton (1989). The option pricing under the GARCH model have been presented by Duan (1995) and enhanced in literature including Kim *et al.* (2009), and Kim *et al.* (2010). Meanwhile, Buffington and Elliott (2002) applied the regime switching Gaussian process model to option pricing. The option pricing model of Buffington and Elliott (2002) have been enhanced by Liu *et al.* (2006) and Jackson *et al.* (2007). Liu *et al.* (2006) used fast Fourier transform method for option pricing with regime switching model, but their model used the Gaussian process as the driving process. Jackson *et al.* (2007) combined the regime switching model with Lévy driving processes, and they numerically solved a partial integro differential equation with the Fourier space time stepping for computing European option prices.

In this paper we will present the hybrid option pricing model combining the regime switching model with the tempered stable driving process. We refer to the hybrid model as the regime switching tempered stable model. Different from Jackson *et al.* (2007), We will use the fast Fourier transform method for calculating European option prices.

The remainder of this paper is organized as follows: In Section 2, we review the regime switching model and the tempered stable process. The regime switching tempered stable model is present in Section 3. Section 4 provides the calibration result of the regime switching tempered stable model for the S&P 500 index option. Finally, Section 5 is a summary of our conclusions.

2 Regime Switching Model

We suppose the economic state of the world is described by a finite state Markov chain and consider a risky asset with volatility depending on the state of the economy. The state space of economy is denoted as $E = (E_t)_{t\geq 0}$ with possible outcomes $\{e_1, e_2, ..., e_N\}$, where $e_k = (0, ..., 0, 1, 0, ..., 0)' \in R^N$ and the process E is time-homogeneous and has a generator matrix Λ. The volatility is denoted as $\sigma = \{\sigma_1, \sigma_2, ..., \sigma_N\}$ with respect to the economic outcomes, where N is the number of states. Thus, the instantaneous volatility can be written as:

$$\sigma_t = \langle \sigma, E_t \rangle, \quad 0 \leq t \leq T \tag{1}$$

where T is the maturity time of the risky asset. Because the Markov process has a countable state space, the amount of time spending on each state e_k for $1 \leq k \leq N$ from 0 to time T is given as:

$$T_k = \int_0^T \langle \sigma_k, E_u \rangle du, \;\; T_k \in \{T_1, T_2, ..., T_N\}, \tag{2}$$

where $\sum_{k=1}^N T_k = T$. The joint distribution of the random vector $(T_1, T_2, ..., T_{N-1})$ is defined by the characteristic function

$$E\left[\exp\left(i\sum_{k=1}^{N-1}\theta_k T_k\right)\right] = \langle \exp[(\Lambda + i\mathrm{diag}\theta)T]E_0, \mathbf{1}\rangle^1. \tag{3}$$

where $\theta = (\theta_1, \theta_2, ..., \theta_{N-1}, 0) \in R^N$, $\mathbf{1} = (1, 1, ..., 1)' \in R^N$, and E_0 is the initial state.

3 Regime Switching Tempered Stable Model

An infinitely divisible distribution is called *Classical Tempered Stable* (CTS) distribution with parameters $(\alpha, C, \lambda_+, \lambda_-, m)$, if its characteristic function is given by

$$\begin{aligned}\phi_{CTS}(u) = \exp(ium - iuC\Gamma(1-\alpha)(\lambda_+^{\alpha-1} - \lambda_-^{\alpha-1}) \\ + C\Gamma(-\alpha)((\lambda_+ - iu)^\alpha - \lambda_+^\alpha + (\lambda_- + iu)^\alpha - \lambda_-^\alpha)).\end{aligned}$$

When a random variable X is CTS distributed with parameter $(\alpha, C, \lambda_+, \lambda_-, m)$, we denote

$$X \sim \mathrm{CTS}(\alpha, C, \lambda_+, \lambda_-, m),$$

Moreover, a Lévy process generated by the CTS distribution is the CTS process with parameter $(\alpha, C, \lambda_+, \lambda_-, m)$. If $C = (\Gamma(2-\alpha)(\lambda_+^{\alpha-2} + \lambda_-^{\alpha-2}))^{-1}$ and $m = 0$, then the CTS distribution has unit variance and zero mean. In this case the CTS distribution is referred to as the standard CTS distribution. Moreover, a Lévy process generated by the standard CTS distribution is referred to as the standard CTS process with parameter $(\alpha, \lambda_+, \lambda_-)$.

Let r is the risk-free rate of return. Assume stock price process S is referred to as the *Regime Switching tempered stable model* if $S = (S_t)_{t\geq 0}$ is given by

$$S_t = S_0 e^{rt} \frac{\exp\left(\int_0^t \langle \sigma, E_s \rangle dZ_s\right)}{E[\exp\left(\int_0^t \langle \sigma, E_s \rangle dZ_s\right)]}$$

where $E = (E_t)_{t\geq 0}$ is the time-homogeneous finite state Markov chain defined in Section 2 and $(Z_t)_{t\geq 0}$ is the standard CTS process with parameters $(\alpha, \lambda_+, \lambda_-)$ under the risk neutral measure. Suppose $(\varepsilon_j(\Delta t))_{j=1,2,...,M}$ is a sequence independent and

[1] See the proof in Buffington and Elliott (2002)

identically distributed random variables with

$$\varepsilon_j(\Delta t) \sim \text{CTS}(\alpha, \Delta t C(\alpha, \lambda_+, \lambda_-), \lambda_+, \lambda_-, 0),$$

and $C(\alpha, \lambda_+, \lambda_-) = (\Gamma(2-\alpha)(\lambda_+^{\alpha-2} + \lambda_-^{\alpha-2}))^{-1}$ under $\mathbf{Q}$. Then we have

$$Y_T = \int_0^T \langle \sigma, E_t \rangle dZ_t = \lim_{M\to\infty} \left(\sum_{j=0}^{M-1} \langle \sigma, E_{t_j} \rangle \varepsilon_j(\Delta t) \right),$$

with $\Delta t = T/M$ and $t_j = \Delta t \cdot j$. By the property of the hidden Markov chain process, we have

$$Y_T = \lim_{M\to\infty} \left(\sum_{k=1}^{N-1} (\sigma_k - \sigma_N) X(M_k) + \sigma_N X(M_N) \right),$$

where $M_k = \#\{t_j | \langle \sigma, E_{t_j} \rangle = \sigma_k, j = 0, \cdots, M-1\}$ and $X(M_k) = \sum_{j=1}^{M_k} \varepsilon_j$ for $k = 1, 2, \cdots, N$. Since we have

$$X(M_k) \sim CTS(\alpha, T_k^M C(\alpha, \lambda_+, \lambda_-), \lambda_+, \lambda_-, 0),$$

where $T_k^M = M_k \cdot \Delta t$, the characteristic function $X(M_k)$ is equal to

$$\phi_{X(M_k)}(u) = E[e^{iuX(M_k)}] = \exp(T_k^M \psi(u; \alpha, \lambda_+, \lambda_-))$$

where

$$\begin{aligned}\psi(u; \alpha, \lambda_+, \lambda_-) = &-iuC(\alpha, \lambda_+, \lambda_-)\Gamma(1-\alpha)(\lambda_+^{\alpha-1} - \lambda_-^{\alpha-1}) \\ &+ C(\alpha, \lambda_+, \lambda_-)\Gamma(-\alpha)((\lambda_+ - iu)^\alpha - \lambda_+^\alpha + (\lambda_- + iu)^\alpha - \lambda_-^\alpha).\end{aligned}$$

Moreover, we have

$$\lim_{M\to\infty} T_k^M = T_k = \int_0^T \langle \sigma_k, E_u \rangle du, \ \ k = 1, \cdots, N,$$

and hence

$$\lim_{M\to\infty} \phi_{X(M_k)}(u) = \exp(T_k \psi(u; \alpha, \lambda_+, \lambda_-)), \ \ k = 1, \cdots, N.$$

Therefore we have

$$Y_T = \sum_{k=1}^{N-1} (\sigma_k - \sigma_N) X(T_k) + \sigma_N X(T),$$

where

$$X(t) \sim CTS(\alpha, tC(\alpha, \lambda_+, \lambda_-), \lambda_+, \lambda_-, 0).$$

The characteristic function of Y_T is equal to

$$\begin{aligned}\phi_{Y_T}(u) &= E\left[\exp\left(iu\sum_{k=1}^{N-1}(\sigma_k-\sigma_N)X(T_k)+iu\sigma_N X(T)\right)\right]\\ &= E\left[\exp\left(\sum_{k=1}^{N-1}\theta_k(u)T_k\right)\right]\exp\left(T\psi(\sigma_N u;\alpha,\lambda_+,\lambda_-)\right),\end{aligned}$$

where $\theta_k(u)=\psi((\sigma_k-\sigma_N)u;\alpha,\lambda_+,\lambda_-)$. By (3), we have

$$\phi_{Y_T}(u)=\langle\exp[(\Lambda+i\mathrm{diag}(\theta(u)))T]E_0,\mathbf{1}\rangle\cdot\exp(T\psi(\sigma_N u)), \tag{4}$$

where $\theta(u)=(\theta_1(u),\cdots,\theta_{N-1}(u),0)$.

4 Calibration of Regime Switching Tempered Stable Model to the S&P 500 Index Option

In order to see whether the new model can survive from the volatile market, the data set[2] of option prices is used from S&P 500 index at the close of market on September 15, 2008 on which Lehman Brothers announced bankruptcy, and we also compare the new model with CTS model and standard Black-Scholes Model. The 13-week Treasury bill index (IRX) is used for risk-free rate of return. The dividend of the S&P 500 is not considered. Due to the call-put parity, we only test the model with call option, which has its prices between \$5 and \$180, and $0.8\le M\le 1.2$, where M is moneyness defined by $M=K/S_0$. The day-to-maturity, which does not include weekends, is between 5 days and 150 days.

To calculate the European option, we use the Fourier transform method presented by Carr and Madan (1999) and Lewis (2001). Under the Fourier transform method, the European call option pricing formula is given by

$$C_t=\frac{K^{1+\rho}e^{-r(T-t)}}{\pi S_t^{\rho}}\Re\int_0^\infty e^{-iu\log(K/S_t)}\frac{\Phi(u+i\rho)}{(\rho-iu)(1+\rho-iu)}du, \tag{5}$$

where $\Phi(u)$ is the characteristic function of $\log(S_T/S_t)$ and ρ is real number such that $\rho<-1$ and $\Phi(u+i\rho)<\infty$ for all $u\in\mathbf{R}$.

Additionally, we assume that $\rho=-1.75$ and initial state is $X_0=e_2=\{0,1,0\}$, so there are three states that the volatility can switch in the Markov chain. If the middle-state volatility is set to σ_2, then the other two volatilities are assumed to be $\sigma_1=0.75\sigma_2$ and $\sigma_3=1.25\sigma_2$, respectively, such as,

$$\Lambda=\left(\begin{array}{c|ccc}\sigma_1 & -\lambda_m & \lambda_m & 0\\ \sigma_2 & \lambda_m & -2\lambda_m & \lambda_m\\ \sigma_3 & 0 & \lambda_m & -\lambda_m\end{array}\right), \tag{6}$$

[2] The data are obtained from Option Metrics's IvyDB in the Wharton Research Data Services.

The calibration parameters are shown in the Table 1. In the table, we also present calibrated parameters for the Black-Scholes model and the *exponential CTS model*. The stock price process $S=(S_t)_{t\geq 0}$ of the exponential CTS model is defined by $S_t = S_0 e^{rt+X_t}/E[e^{X_t}]$ for all $t \geq 0$ where the process $(X_t)_{t\geq 0}$ is the CTS process with parameter $(\alpha, C, \lambda_+, \lambda_-, 0)$. The call option price in the exponential CTS model is also obtained by (5).

To evaluates the performance of three models, we use four error estimators: average prediction error (APE), average absolute error (AAE), root mean-square error (RMSE), and average relative pricing error (ARPE).[3] The four error estimators are shown in Table 1. By the table, we observe that the regime switching tempered stable model has the smallest errors. The implied volatility curves for the three models are presented in Figure 1. This figure shows that the regime switching tempered stable model matches the market price better than exponential CTS model, in this investigation.

5 Conclusion

In this talk we have discussed the regime switching model with the tempered stable driving process together with option pricing. Fast Fourier transform algorithm is playing an advance role in computing option prices under the model. The characteristic function of time occupation in the regime switching model also shows its potential ability to implement in the integration of a wide range of other models. Furthermore, the calibration results for this new model are considered with satisfactory accuracy.

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[3] Let N be the number of observation, P_n be the n-th observed price, and $\hat{P}_n$ be the n-th price determined by the option price formula. Then the four error estimators are defined as follows: $\text{AAE} = \sum_{n=1}^{N} \frac{|P_n - \hat{P}_n|}{N}$, $\text{APE} = \text{AAE}\left(\sum_{n=1}^{N} \frac{P_n}{N}\right)^{-1}$, $\text{ARPE} = \frac{1}{N}\sum_{n=1}^{N} \frac{|P_n - \hat{P}_n|}{P_n}$, $\text{RMSE} = \sqrt{\sum_{n=1}^{N} \frac{(P_n - \hat{P}_n)^2}{N}}$.

Table 1 Results for the calibration of the risk-neutral parameters on September 15, 2008

τ	N	Model	Parameters			AAE	APE	ARPE	RMSE
24	22	BS	$\sigma = 0.2791$			1.5871	0.0837	0.1307	1.8090
		Exp-CTS	$\alpha = 1.2369$	$C = 0.4893$		1.7411	0.0919	0.1497	1.9849
			$\lambda_+ = 221.2633$	$\lambda_- = 17.3431$					
		RS-CTS	$\sigma = 0.2558$	$\lambda_m = 19.2961$		0.2783	0.0147	0.0218	0.3154
			$\alpha = 0.2000$	$\lambda_+ = 129.6013$	$\lambda_- = 4.6660$				
49	13	BS	$\sigma = 0.2539$			2.5459	0.1018	0.1727	2.8491
		Exp-CTS	$\alpha = 1.2056$	$C = 0.4837$		2.0913	0.0836	0.1393	2.2861
			$\lambda_+ = 224.2847$	$\lambda_- = 18.7186$					
		RS-CTS	$\sigma = 0.2467$	$\lambda_m = 199.9997$		0.2737	0.0109	0.0145	0.3635
			$\alpha = 0.2000$	$\lambda_+ = 19.4587$	$\lambda_- = 2.5297$				
69	17	BS	$\sigma = 0.2512$			2.5729	0.0618	0.1251	2.9650
		Exp-CTS	$\alpha = 1.1490$	$C = 0.4860$		1.7444	0.0419	0.0831	1.9861
			$\lambda_+ = 228.5896$	$\lambda_- = 14.5285$					
		RS-CTS	$\sigma = 0.2513$	$\lambda_m = 2.9433$		0.2066	0.0050	0.0080	0.3691
			$\alpha = 0.7815$	$\lambda_+ = 21.0696$	$\lambda_- = 1.0962$				

τ:Day-to-maturity BS: Black-Scholes model
N:Number of options Exp-CTS: Exponential CTS model
RS-CTS: Regime switching tempered stable model

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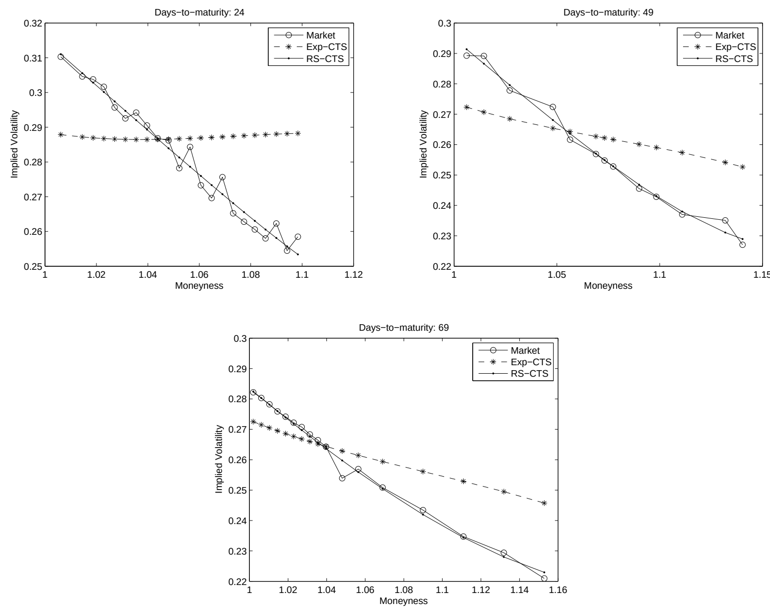
Fig. 1 Implied Volatility on September 15, 2008



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