

Approximate truth of perfectness - an experimental test

by Siegfried Berninghaus, Werner Güth, King
King Li

No. 41 | JUNE 2012

WORKING PAPER SERIES IN ECONOMICS



Impressum

Karlsruher Institut für Technologie (KIT)
Fakultät für Wirtschaftswissenschaften
Institut für Wirtschaftspolitik und Wirtschaftsforschung (IWW)
Institut für Wirtschaftstheorie und Statistik (ETS)

Schlossbezirk 12
76131 Karlsruhe

KIT – Universität des Landes Baden-Württemberg und
nationales Forschungszentrum in der Helmholtz-Gemeinschaft

Working Paper Series in Economics
No. 41, June 2012

ISSN 2190-9806

econpapers.wiwi.kit.edu

Approximate Truth of Perfectness

- An Experimental Test -

Siegfried Berninghaus, Werner Güth, King King Li

May 2012

Abstract

“Approximate truth” refers to the principle that border cases should be analyzed by solving generic cases and solving border cases as limits of generic ones (Brennan et al., 2008). Our study experimentally explores whether this conceptual principle is also behaviorally appealing. To do so, we focus on perfectness (Selten, 1975) and use his example game with (no) multiplicity of (perfect) equilibria. Distinguishing three uniform perturbation levels, we check for monotonicity (all players react monotonically to the perturbation level) and then explore the behavioral relevance of “approximate truth.”

Keywords: experimental games, trembling hand perfectness, perturbed strategies

JEL classification: C70, C72, C91

1. Introduction

In economics but also in its neighboring disciplines, there is a long tradition of solving ideal cases directly rather than by first solving generic cases and then solving the ideal or border cases by determining the limits of solutions of generic ones (Brennan, Güth, and Kliemt, 2008). This principle is more or less explicitly propagated by Harsanyi (1973), Selten (1975), and Harsanyi and Selten (1988). Examples of studies violating this principle are discussed in Brennan et al. (2008), whose list is, of course far from being exhaustive. Here we focus on the ideal case of unperturbed games and perfectness as a requirement of approximate truth (Selten, 1975).

In an unperturbed game, players can perfectly realize their intended activities, i.e., an unintended action is impossible. The consequence of such ideal power of command are unreached information sets in extensive form games or, analogously, impossible strategy vectors in normal form games. In a generic, i.e., perturbed game, the power to command one's actions is limited: for all players any action has possibly a very small, but strictly positive probability of being used unintentionally.

This obviously avoids unreached information sets in extensive form games and ensures that Bayes' rule (see Bayes, 1763) is always applicable. Similarly, all possible strategy vectors are realized with positive probability in normal form games. A specific consequence of the latter phenomenon is that whatever strategy is weakly dominated in the unperturbed game will be strictly dominated in a perturbed game. This obviously eliminates equilibria in weakly dominated strategies as candidates for perfect or "approximately true" equilibria.¹

We experimentally implement Selten's (1975) example of a three-person game with multiple equilibria only one of which is perfect, however. In line with an abstract solution requirement² (see Harsanyi and Selten, 1988), and to avoid demand effects of asymmetric minimum choice probabilities we induce constant minimum choice probability for all choices of all players. We thus explore the "approximate truth" of uniformly perfect equilibria by exogenously imposing constant minimum choice probabilities which vary between treatments.

In section 2 we describe the experimental protocol. Section 3 analyzes the experimental data. Section 4 concludes.

¹ In Industrial Organization such equilibria are nevertheless propagated, e.g., as the solution for price competition in asymmetric homogeneous markets with constant unit costs as originally suggested by Bertrand (1883). Here the market is extreme not only in the sense of perfect product homogeneity but also by letting prices vary continuously.

² By denying any rationality in making mistakes (see, e.g., Myerson, 1978, who postulates such rationality), all actions should have the same positive minimum probability of being used.

2. Experimental Protocol

Our experimental workhorse is the three-person game in Figure 1 (see Selten, 1975) with players $i = 1, 2, 3$ who have to decide between R_i and L_i . Since R_3 is weakly dominated, the equilibrium $R = (R_1, R_2, R_3)$ is not perfect in spite of its attractivity (R is, e.g., payoff undominated). Thus the “approximately true” solution is the perfect equilibrium $L = (L_1, L_2, L_3)$.

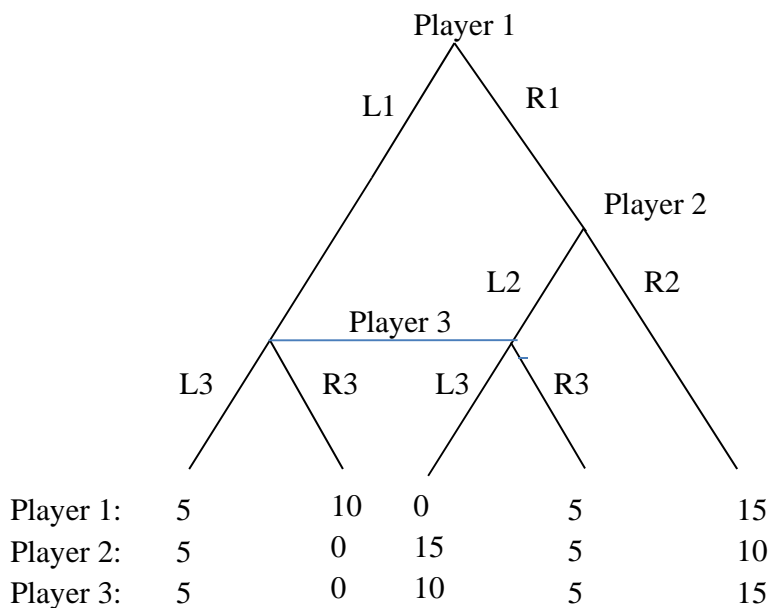


Figure 1: The extensive form game with players $i = 1, 2, 3$ and choices L_i and R_i . In the experimental instruction for $i = 1, 2, 3$, the choices are color labeled. By connecting the two decision nodes of player 3, it is indicated that they lie in one information set.

Participants were randomly and anonymously matched into groups of three participants to play the game in Figure 1. Player 1 (2, 3) determines the probability that the play will proceed with

move L_1 (L_2 , L_3) respectively R_1 (R_2 , R_3). For $i = 1, 2, 3$, the probability for L_i , respectively R_i is determined by the following urn: in the 25 free balls game with altogether 27 balls, only one (of them) is predetermined as red (for L_i) and one as black (for R_i). As for the remaining 25 balls, player i can freely determine how many of them are red or black. Once the urn is composed by determining the color of all 27 balls, a ball will be randomly drawn from it. If a red ball is drawn, the game will proceed with route L_i . If a black ball is drawn, the game will proceed with route R_i . Similarly, in the 21 free balls game, there are 3 red balls, and 3 predetermined black balls, and player i determines the color of the remaining 21 free balls. In the 9 free balls game, 9 red balls and 9 black balls have a predetermined color, meaning that player i determines the color of only 9 balls.

Note that even in the 9 free ball game, the avoidance of R_3 as far as possible, i.e., player 3 plays L_3 with probability $2/3$ and R_3 with probability $1/3$, suffices to render L_2 uniquely optimal for player 2 (since L_2 yields $35/3$ and R_2 only 10). Anticipating that both, player 2 and player 3, use their L -strategy with maximal probability $2/3$ in the 9 free balls game, finally implies that for player 1 the choice of L_1 , yielding $60/9$, is better than R_1 , yielding only $55/3$. Thus even the largest uniform perturbation level allows for no ambiguity of (sequential) rationality. For all perturbation levels all three players $i = 1, 2, 3$ should therefore use the perfect equilibrium strategy L_i with maximal probability.

For the sake of more informative data, and to encourage more thorough deliberations on what to do in the various roles (1, 2, and 3), choices were elicited using the strategy vector method. That is, participants specified their choice s_1, s_2 , and s_3 via determining the urns with red and black balls to govern the choice of player 1, 2, and 3. Given these choices, the computer randomly determined the roles, implemented the decisions, and computed the payoffs accordingly. For example, if L_1 and L_3 are realized, each player receives 5 euros.

To investigate within-subject differences across treatments, subjects participated in all three games, differing only in their uniform trembles where the order was varied between subjects. The three treatments can be distinguished by the sequence of the numbers of “free balls”:

<i>Treatments</i>		
25-21- 9	21-21-21	9-21-25

For example, in the 25-21-9 treatment, subjects first played the 25 free balls game, then the 21 free balls game, and finally the 9 free balls game. Subjects were not informed that they would play the second (third) game when they played the first (second) game in the sequence. They learned about payoffs only after playing all three games and additionally reported their beliefs concerning the choice behavior of others.

A total of 135 subjects participated in the computerized (via z-tree, Fischbacher, 2007) experiment. They were randomly recruited from a poll of approximately 2,500 subjects using an e-mail recruitment system (ORSEE, see Greiner, 2004). All subjects were university students from Jena, Germany. The experiments were conducted in German. In the beginning, subjects were given paper instructions. They were informed that their decisions would be anonymous and that they would receive a show-up fee of 2.5 euros in addition to what they earned by playing the games. Then each subject was asked to key in her decisions.

3. Experimental Results

Let us begin by analyzing first-play behavior, i.e., we use the combined first choices

- of “25 free balls” from treatment 25-21-9
- of “21 free balls” from treatment 21-21-21
- of “9 free balls” from treatment 9-21-25

where we concentrate on the proportion of “free balls” allocated to R_i and L_i , respectively. Since only the equilibrium $L = (L_1, L_2, L_3)$ is perfect, we check monotonicity by testing

Hypothesis M: Monotonic convergence to L .

For $i = 1, 2, 3$ the tendency to voluntarily use L_i increases with more “free balls.”

Table 1. Average probability allocated

Treatment	Average Number of Allocated Balls (Std.)		
	L1	L2	L3
25 Free Balls	5.63 (9.01)	5.30 (8.29)	20.81 (6.27)
21 Free Balls	5.80 (8.43)	11.47 (9.79)	19.03 (4.73)
9 Free Balls	3.00 (3.34)	2.47 (3.25)	7.87 (2.11)
Treatment	Average Probability Allocated (Std.)		
	L1	L2	L3
25 Free Balls	0.25 (0.33)	0.23 (0.31)	0.81 (0.23)
21 Free Balls	0.33 (0.31)	0.54 (0.36)	0.82 (0.18)
9 Free Balls	0.44 (0.12)	0.42 (0.12)	0.62 (0.08)

Notes: 25 Free Balls refers to the first round in the 25-21-9 treatment. 21 Free Balls refers to the first round in the 21-21-21 treatment. 9 Free Balls refers to the first round in the 9-21-25 treatment.

Table 1 shows voluntary probability of L_i as depending on the number of “free balls,” separately for each player $i = 1, 2, 3$.

Clearly, the weakly dominating move L_3 is realized with higher probability than L_1 and L_2 , which could be expected as the non-rationality of L_3 is more obvious. Concerning the effect of variation in the number of free balls, the change from 25 to 21 “free balls” is minor, compared to the one from 25 or 21 to 9 “free balls.” The intermediate treatment of “21 free balls” was mainly introduced to check only minor “perturbance differences.” As can be seen from Table 1, due to noisy choice behavior, one cannot expect monotone reactions to minor changes in the perturbance level. However, the differences between 25 or 21 “free balls” and 9 “free balls” are statistically significant for all three player roles, see Table 2.³

Table 2. Comparisons of Probability Allocated for L

Average Probability Allocated for L	Mean Differences	t-Test
L1		
25 vs. 9	0.20	3.04***
21 vs. 9	0.12	1.93*
L2		
25 vs. 9	0.19	3.16***
21 vs. 9	-0.11	-1.59
L3		
25 vs. 21	-0.18	-4.08***
21 vs. 9	-0.19	-5.46***

Notes: *, **, and *** represent significance at 10, 5, and 1 percent levels, respectively.

The average probabilities observed in 25 “free balls” for L_1 and L_2 are significantly lower than in the 9 “free balls,” whereas the reverse is observed with L_3 . We thus can conclude the following:

Observation 1: When the number of “free balls” decreases, the voluntary probabilities of relying on the perfect equilibrium strategies L_1 and L_2 increases where the effects when comparing 25 or 21 “free balls” with 9 “free balls” are statistically significant. For L_3 an opposite effect is observed.

³ Except the case of 21 vs. 9 in L2.

The observation for L_1 and L_2 contradicts hypothesis M, whereas the observation for L_3 is in line with hypothesis M. As in every perturbed game R_3 is strictly dominated, one might expect the strongest avoidance of R_3 , whose non-optimality is more obvious. The probability allocated for L_3 is significantly higher than L_1 and L_2 (p-value = 0.00). There is no significant difference between L_1 and L_2 .

Observation 2: For all numbers of “free balls” player 3 avoids the dominated choice R_3 significantly more often than the two other players, 1 and 2, for whom the incentive to avoid R_i anticipates that player 3 avoids R_3 to such an extent that L_2 and then also L_1 become optimal.

Let us finally investigate how earlier experience with another “perturbance” level affects behavior. Table 3 below tabulates the probability allocated by players in respective treatments. Contrary to the expectation that any earlier experience should strengthen the tendency of voluntarily using L_i rather than R_i , $i = 1, 2, 3$, there is no significant difference.

Observation 3: When for $i = 1, 2, 3$ comparing the first and third play

- of 25 “free balls” (i.e., using the 1st (3rd) play data of treatments 25-21-9 (9-21-25), the use of L_i is not significantly stronger,
- of 9 “free balls” (i.e., using the 1st (3rd) play data of treatments 9-21-25 (25-21-9), the use of L_i is not significantly stronger.

Table 3. The Effect of Sequence of Play on Probability Allocated

	Probability Allocated			
	25 Free Balls		9 Free Balls	
	25-21-9	9-21-25	25-21-9	9-21-25
Player 1	0.25 (0.06)	0.27 (0.06)	0.43 (0.03)	0.44 (0.02)
Player 2	0.23 (0.06)	0.39 (0.07)	0.40 (0.02)	0.42 (0.02)
Player 3	0.81 (0.04)	0.85 (0.05)	0.62 (0.02)	0.62 (0.01)

Observation 3 that former experiences do not strengthen the use of perfect equilibrium strategies is additionally supported by the 21-21-21-treatment.

Observation 4: The voluntary probabilities of using L_i rather than R_i does not increase significantly with the round of play of the 21-21-21-treatment, even when the game, captured by the noise level, remains constant.

Table 4. Probability Allocated to L_i in Treatment 21-21-21

	First Round	Second Round	Third Round
Player 1	0.33 (0.06)	0.41 (0.06)	0.44 (0.06)
Player 2	0.54 (0.07)	0.46 (0.06)	0.52 (0.06)
Player 3	0.82 (0.03)	0.80 (0.03)	0.78 (0.03)

4. Conclusion

It would have been naïve to expect that the unique perfect equilibrium prediction could be experimentally confirmed. One might have hoped that less perturbation would shift behavior toward its benchmark. But this could be confirmed only for player 3 whose behavior is much closer to the predicted choice (of L_3) but not for player 1 and player 2. In our view, the non-optimality of L_3 is much more obvious than that of L_1 and L_2 rendering the choices by players 1 and 2 much more noisy. This explains that minor changes in perturbation (21 versus 25 “free balls”) have little effect and that the use of L_1 respectively L_2 is far from being close to what was predicted. Altogether, we have not consistently proved that the principle of approximate truth is in line with the experimental evidence collected. Equilibrium refinements relying on this principle are therefore only philosophically but not psychologically appealing.

One may object to our analysis that we assume common(ly known) material opportunism in the sense that each player only cares for his own monetary payoff expectation. Since the imperfect equilibrium R payoff dominates the perfect equilibrium L , assuming efficiency concerns would allow to justify R rather than L as solution. However, this would require some common knowledge of such “social preferences,” which seems highly unrealistic. But the fact that R payoff dominates L means that our experimental game is a worst-case scenario for confirming the principle of approximate truth. Across the board, our results indicate that the principle obviously does not have behavioral appeal. If at all, it needs more favorable conditions than those provided by our experimental game.

References

- Bayes, T. (1763). An Essay towards solving a Problem in the Doctrine of Chances. *Philosophical Transactions of the Royal Societies of London*, 53, 370-418.
- Bertrand, J. (1883). Théorie Mathématique de la Richesse Sociale. *Journal des Savants*, 499-508.
- Brennan, G., Güth, W., & Kliemt, H. (2008). Approximate truth in economic modelling. *Homo Oeconomicus* 25 (3/4), 1-20.
- Fischbacher, U. (2007). z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics* 10(2):171-178.
- Greiner, B. (2004). The online recruitment System ORSEE 2.0 - A guide for the organization of experiments in economics, *Working Paper Series in Economics 10*, University of Cologne.
- Harsanyi, J. C. (1973). Games with randomly disturbed payoffs: a new rationale for mixed-strategy equilibrium points. *International journal of game theory*, 2(1), 1-23.
- Harsanyi, J. C., & Selten, R. (1988). *A general theory of equilibrium selection in games*. Cambridge, MA: The MIT Press.
- Myerson, R.B. (1978). Refinements of the Nash equilibrium concept. *International Journal of Game Theory*, 15, 133-154.
- Selten, R. (1975). Reexamination of the perfectness concept for equilibrium points in extensive games. *International Journal of Game Theory*, 4(1), 25-55.

Appendix

We present the experimental instructions for one urn experiment only (with 25 free balls). It is easy to reconstruct the experimental instructions for the remaining cases.

Experimental Instructions (25 free balls)

Welcome to our experimental study on decision-making. You will receive a show-up fee of Euro2.5. In addition, you can gain more money as a result of your decisions in the experiment.

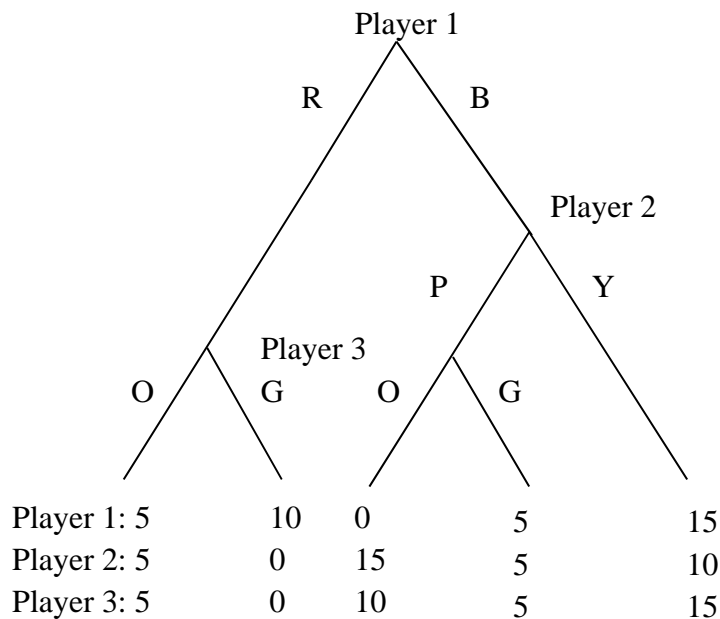
You will be given a subject ID number. Please keep it confidentially. Your decisions will be anonymous and kept confidential. Thus, other participants won't be able to link your decisions with your identity. You will be paid in private, using your subject ID, and in cash at the end of the experiment.

When you have any questions, please feel free to ask by raising your hand, one of our assistants will come to answer your questions. Please DO NOT communicate with any other participants.

The Game

You will be randomly and anonymously paired with two other participants to play the following game. In this game, there are three players: player 1, player 2, and player 3. You will need to make your decisions under each role. More specifically, you will need to specify your decisions as if you are player 1, as if you are player 2, and as if you are player 3. In the end of the experiment, the computer will randomly determine your role, implement your decisions under that role, and pay you accordingly.

The game has three stages.



Stage 1

Player 1 makes a choice which will determine the probability that the game will proceed with route R or B. There are 27 balls, and 1 of them is red and 1 of them is black. For the remaining 25 balls, player 1 will determine the number of red and black balls, i.e., how many of the remaining 25 balls will be red or black. Then a ball will be randomly drawn from the urn with altogether 27 red and black balls. If a red ball is drawn, the game will proceed with route R. If a black ball is drawn, the game will proceed with route B.

Stage 2

Player 2 makes a choice which will determine the probability that the game will proceed with route P or Y. There are 27 balls, and 1 of them is pink and 1 of them is yellow. For the remaining 25 balls, player 2 will determine the number of pink and yellow balls, i.e., how many of the remaining 25 balls will be pink or yellow. Then a ball will be randomly drawn from the urn with altogether 27 pink and yellow balls. If a pink ball is drawn, the game will proceed with route P. If a yellow ball is drawn, the game will proceed with route Y. Player 2 will make the decision without knowing the color of the ball drawn in stage 1.

Stage 3

Player 3 makes a choice which will determine the probability that the game will proceed with route O or G. There are 27 balls, and 1 of them is orange and 1 of them is grey. For the remaining 25 balls, player 3 will determine the number of red and black balls i.e., how many of the remaining 25 balls will be orange or grey. Then a ball will be randomly drawn from the urn with altogether 27 orange and grey balls. If an orange ball is drawn, the game will proceed with route O. If a grey ball is drawn, the game will proceed with route G. Player 3 will make the decision without knowing the color of the ball drawn in stage 1 and stage 2.

Payoffs

If route R and route O are implemented, each player will receive 5 Euro.

If route R and route G are implemented, player 1 will receive 10 Euro. Player 2 and 3 will both receive 0 Euro.

If route B, route P, and route O are implemented, player 1 will receive 0 Euro, while player 2 will receive 15 Euro, and player 3 will receive 10 Euro.

If route B, route P, and route G are implemented, each player will receive 5 Euro.

If route B and route Y are implemented, player 1 will receive 15 Euro, while player 2 will receive 10 Euro, and player 3 will receive 15 Euro.

Please decide now!

If you are player 1

If I am player 1, I want _____ of the remaining balls to be red and _____ balls be black.

If you are player 2

If I am player 1, I want _____ of the remaining balls to be pink and _____ balls be yellow.

If you are player 3

If I am player 1, I want _____ of the remaining balls to be orange and _____ balls be black grey.

Questionnaire

Now we have some questions for you. Please answer them carefully. Your answers will not influence your final payoff.

1. In your estimation, how many percent of player 1 (other than yourself) have chosen to allocate a positive number of black balls?

_____ %

2. In your estimation, what is the average number of black balls (out of 25) chosen by other participants (player 1)?

_____ black balls

3. In your estimation, how many percent of player 2 (other than yourself) have chosen to allocate a positive number of yellow balls?

_____ %

4. In your estimation, what is the average number of yellow balls (out of 25) chosen by other participants (player 2)?

_____ yellow balls

5. In your estimation, how many percent of player 3 (other than yourself) have chosen to allocate a positive number of grey balls?

_____ %

6. In your estimation, what is the average number of grey balls (out of 25) chosen by other participants (player 3)?

_____ grey balls

Working Paper Series in Economics

recent issues

- No. 41** *Siegfried Berninghaus, Werner Güth, King King Li: Approximate truth of perfectness - an experimental test, June 2012*
- No. 40** *Marten Hillebrand and Tomoo Kikuchi: A mechanism for booms and busts in housing prices, May 2012*
- No. 39** *Antje Schimke: Entrepreneurial aging and employment growth in the context of extreme growth events, May 2012*
- No. 38** *Antje Schimke, Nina Teichert, Ingrid Ott: Impact of local knowledge endowment on employment growth in nanotechnology, February 2012*
- No. 37** *Siegfried K. Berninghaus, Lora Todorova, Bodo Vogt: A simple questionnaire can change everything - are strategy choices in coordination games stable?, December 2011*
- No. 36** *Matthias Duschl, Antje Schimke, Thomas Brenner, Dennis Luxen: Firm growth and the spatial impact of geolocated external factors - empirical evidence for German manufacturing firms, November 2011*
- No. 35** *Mirja Meyborg: The impact of West-German universities on regional innovation activities - a social network analysis, October 2011*
- No. 34** *Tom Broekel, Antje Schimke, Thomas Brenner: The effect of cooperative R&D subsidies and subsidized cooperation on employment growth, October 2011*
- No. 33** *Stephan Schosser and Bodo Vogt: The public loss game - an experimental study of public bads, August 2011*
- No. 32** *Antje Schimke and Thomas Brenner: Temporal structure of firm growth and the impact of R&D, July 2011*
- No. 31** *Axel Schaffer: Appropriate policy measures to attract private capital in consideration of regional efficiency in using infrastructure and human capital, July 2011*