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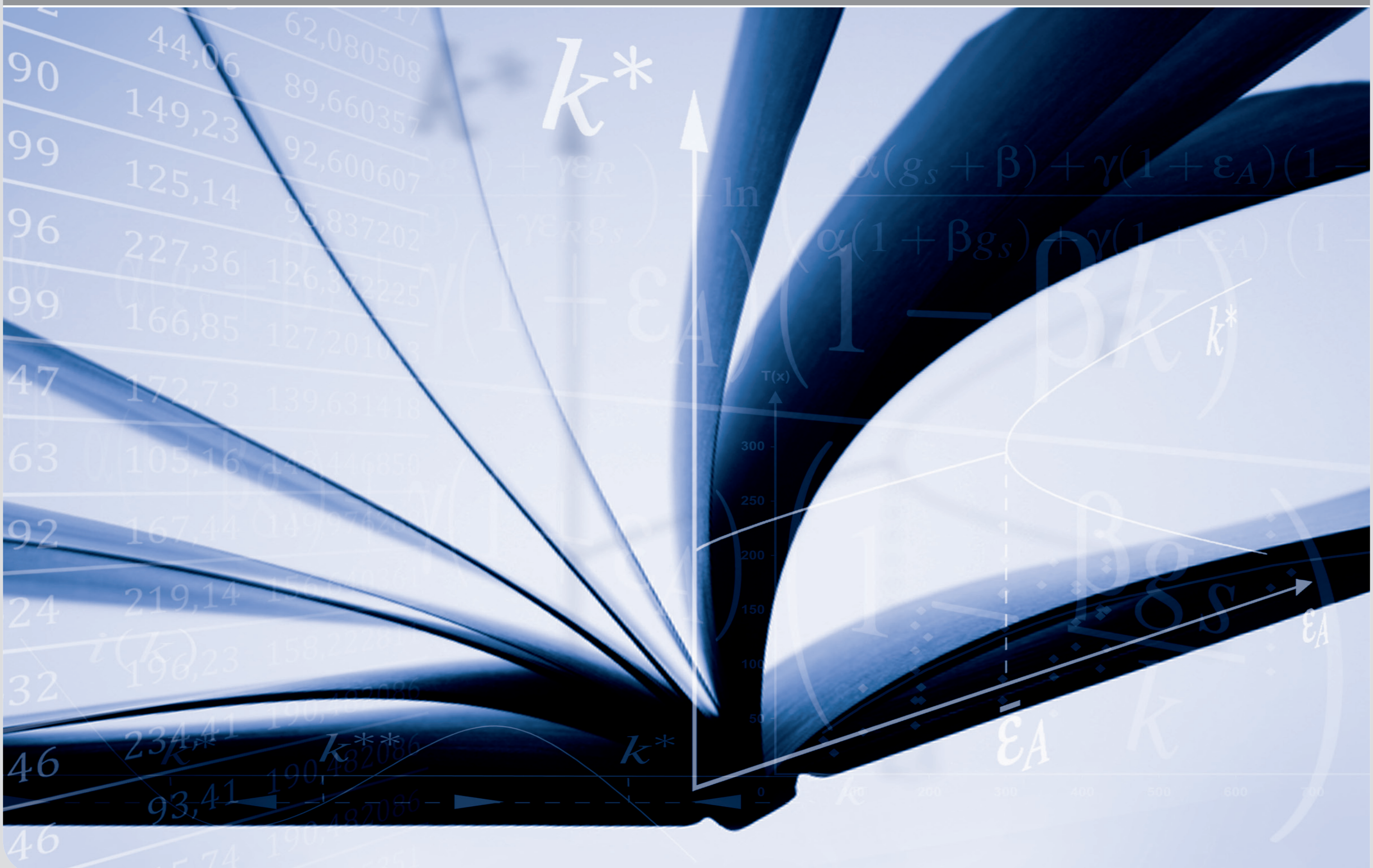

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LEARNING IN NETWORKS – AN EXPERIMENTAL STUDY USING STATIONARY CONCEPTS

Siegfried K. Berninghaus[a], Thomas Neumann[b, 1] and Bodo Vogt[b]

[a] Karlsruhe Institute of Technology (KIT), Institute for Economic Theory and Statistics, P.O. Box 6980, D-76128 Karlsruhe, Germany

[b] Otto-von-Guericke-University Magdeburg, Faculty of Economics and Management, Empirical Economics, P.O. Box 4120, D-39016 Magdeburg, Germany

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Abstract

Our study analyzes theories of learning for strategic interactions in networks. Participants played two of the 2 x 2 games used by Selten and Chmura (2008) and in the comment by Brunner, Camerer and Goeree (2009). Every participant played against four neighbors and could choose a different strategy against each of them. The games were played in two network structures: a lattice and a circle. We compare our results with the predictions of different theories (Nash equilibrium, quantal response equilibrium, action-sampling equilibrium, payoff-sampling equilibrium, and impulse balance equilibrium) and the experimental results of Selten and Chmura (2008). One result is that the majority of players choose the same strategy against each neighbor. As another result we observe an order of predictive success for the stationary concepts that is different from the order shown by Selten and Chmura. This result supports our view that learning in networks is different from learning in random matching.

Keywords: experimental economics, networks, learning

JEL classification: C70, C73, C91, D83, D85

1. Introduction

In their 2008 paper, Reinhard Selten and Thorsten Chmura (henceforth SC) analyze a set of 12 different 2 x 2 games. For 6 constant and 6 non-constant sum games they compare the predictive success of five stationary concepts. The five concepts compared are: 1. Nash equilibrium (Nash), 2. Quantal response equilibrium (QRE) (McKelvey and Palfrey, 1995), 3. Action-sampling equilibrium (ASE) (Selten and Chmura, 2008), 4. Payoff-sampling equilibrium (PSE) (Osborne and Rubinstein, 1998), and 5. Impulse balance equilibrium (IBE) (Selten and Buchta, 1999, and Selten, Abbink, and Cox, 2005). Since these concepts are explained in detail in SC, we do not explain them here. In their study, the randomly matched participants played the games over 200 rounds, which allows an interpretation of the concepts "... as stationary states of dynamic learning models" (SC, p. 940). In our study we put the focus on learning *in networks*.

[1] Corresponding author: thomas.neumann@ovgu.de.

We recall a point made often and in many situations, that learning occurs in social contexts. When analyzing learning in economic decision-making, most studies deal with repeated interactions between two or more randomly matched players. In real life, learning in networks seems to be more natural. In our study, we will analyze whether learning in networks is different from learning in an environment of random matching. The question of whether subjects actually mix strategies against their network partners is another topic to be investigated in this study. While in a random matching environment players can mix strategies over time, in a network they can mix strategies within one period by playing different strategies against their neighbors. Learning in network structures also occurs via indirect neighbors whose decisions also affect direct neighbors.[2]

From our perspective, the predictions of the five concepts on strategy learning depend neither on the neighborhood nor on the network structure. Therefore, we change the neighborhood from random matching to a network structure to test the impact of this parameter.

In our study, we design a neighborhood game in an exogenously fixed network, where the players cannot choose their neighbors but can choose different strategies against each of the exogenously given neighbors. We run two different games used by SC as baseline games in a network. The experimental results we present allow us to analyze on the one hand how network structures affect learning, and on the other hand how to control for the predictive success of different concepts of learning.

Our analysis is guided by three key questions:

1) Do the participants (actually) use mixed strategies?[3]
2) Does the structure of the network affect learning in the games?
3) Are the learning theories proposed by Selten and Chmura (2008) good predictors for the participants' behavior in network games?

Based on the games used by SC (a constant and a non-constant sum game), we construct neighborhood games, where each player has four direct neighbors. With respect to the idea that players have to decide how they interact with their partners and how they adjust their behavior over time, the participants in our experiment could choose different strategies for each neighbor.

Guided by the findings of Berninghaus, Ehrhart, and Keser (2002), where the players' behavior was affected by different network structures, we used two different structures: a lattice and a circle.

We address our first key question by analyzing how often the players use the ability provided, to play different strategies against their neighbors (to mix strategies within one round).

Comparing the participants' behavior in the two different network structures, we answer our second key question. Additionally, comparing our observed results with those of SC, we want to provide evidence that learning in networks differs from learning in a random matching environment.

To answer our third key question, we compare our experimental results with the predictions of the five learning concepts. The results of SC were revised in studies by Brunner, Camerer, and Goeree (2009) (henceforth BCG) and by Selten, Chmura, and Goerg (2010). To ensure that the results are comparable, we use the same statistical techniques as these studies to analyze our data.

[2] Aspects of network formation, and learning in networks have been the center of attention in many studies. See e.g. Bala and Goyal (2000), Jackson and Watts (2002), and Berninghaus and Vogt (2006).

[3] The use of the term "mixed strategies" is referred to the possibility that players choose a different strategy against each neighbor in one round. In other words, they mix their strategies within this round.

As one of our main results we show that learning in networks is different from learning in random matching. However, we find no significant difference between the two network structures. In line with this result, we observe an order of predictive success for the five concepts that differs from the order given by SC, which means that learning in networks has a significant impact on strategy formation.

As another remarkable result, we find that the majority of players choose the same strategy against each neighbor, i.e. players do not really mix strategies. This holds for both network structures employed in our experiments. Moreover, the average number of players not mixing strategies is slightly higher in the circle network.

2. Description of the experiment

In our study we use two games (see Figure 1) taken from SC. The main reason we use these games is that they generate clearly distinct predictions for the five stationary learning concepts. Figure 2 shows the theoretical predictions of these concepts.

2.1. The baseline games

We use two games, a constant and a non-constant sum game, as baseline games. As explained by SC, both games form a pair that is characterized by having the same best response structure.[4]

10 / 8	0 / 18
9 / 9	10 / 8

10 / 12	4 / 22
9 / 9	14 / 8

Figure 1: The constant and the non-constant sum game

Because we want, on the one hand, to turn the focus to learning by analyzing the impact of different network structures, and on the other hand to refer to the results of SC and BCG, we took the games from SC and extended them to neighborhood games in different network structures.

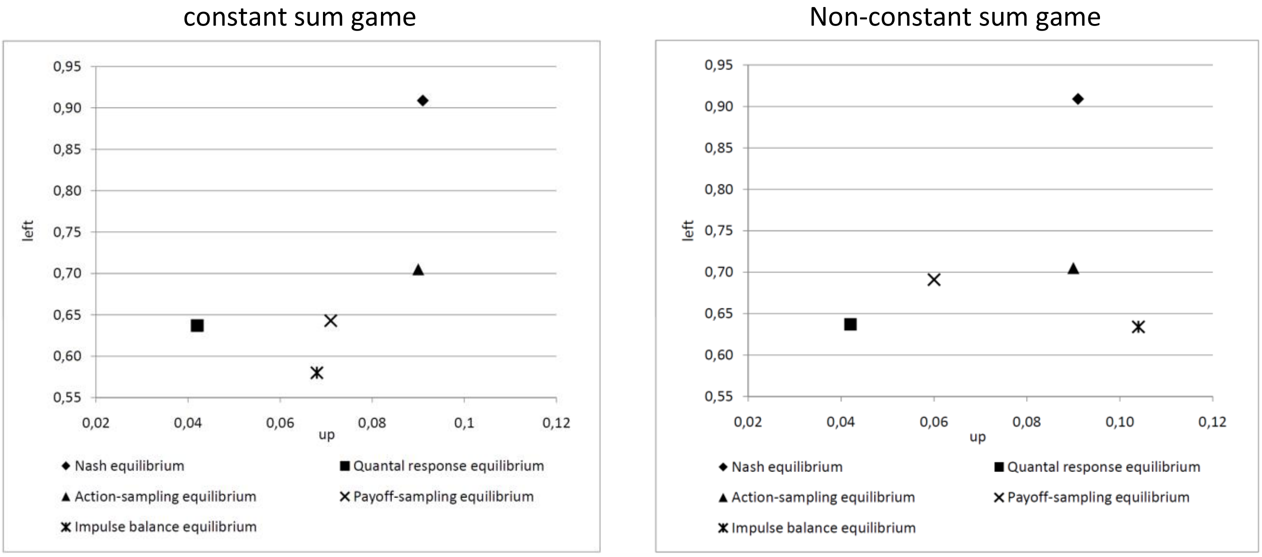


Figure 2: Theoretical equilibria in the games

[4] One can find detailed descriptions of the five concepts and the determination of the equilibria in Selten and Chmura (2008) and in Brunner, Camerer and Goeree (2009).

2.2. *The neighborhood games and network structures*

Following Berninghaus, Ehrhart, and Keser (2002), we ran the games in exogenously given fixed network structures where each player has four direct neighbors. To accomplish this, 16 players were allocated into two different network structures. The 2 x 2 games described in Section 2.1 represent the baseline games. Based on these games, the neighborhood game was constructed such that each player interacts with her four direct (local) neighbors. Suitably to our first key question, in our experiment each player could use a different strategy against each of her neighbors in each round.

Since it is well known in the literature that network structures affect the players' behavior in games (see e.g. Berninghaus, Ehrhart, and Keser (2002), Kirchkamp and Nagel (2007), and Cassar (2007)) we used two different structures. Guided by the findings of Berninghaus, Ehrhart, and Keser (2002) regarding the behavior of players in a coordination game, we used two different network structures: a lattice and a circle. Figure 3 provides a schematic illustration.

Network structure 1 – a lattice　　　　Network structure 2 – a circle

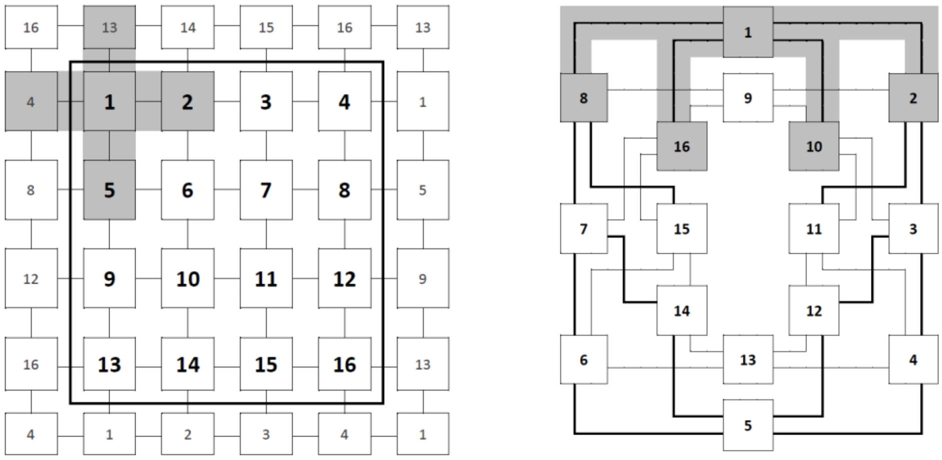


Figure 3: The two network structures

It is obvious that in both structures each player interacts with four direct neighbors. The difference between the structures is given by the number of indirect neighbors on the level of indirect interactions.

There might be more equilibria in the network game as we present in Figure 2. Since we want to compare the players' behaviors in the two different network structures as well as with the experimental results of SC, we use the equilibria presented in Section 2.1 as benchmarks.

Exemplary explanations of the interaction structures:

In the Lattice, player 1 interacts directly with player 2 as her right, player 4 as her left, player 13 as her top, and player 5 as her bottom neighbor[5]. On the first level of indirect interactions, player 1 has six indirect neighbors (players 16, 14, 3, 8, 6, and 9). On the second level of indirect interactions, player 1 has four indirect neighbors (players 15, 7, 12, and 10), and one indirect neighbor (player 11) on the third level of indirect interactions. Behaviors

In the Circle, player 1 interacts directly with player 8 and player 16 as her left neighbors and with player 2 and player 10 as her right neighbors. On the first level of indirect interactions player 1 has

[5] As one can see, players at the edges of the lattice have their neighbors on the opposite edges.

five indirect neighbors (players 9, 7, 15, 11, and 3). On the second level of indirect interactions, player 1 has four indirect neighbors (players 6, 14, 12, and 4), and she has two indirect neighbors (players 13 and 5) on the third level of indirect interactions.

3. Experimental procedure

In order to compare our results with the results of SC, we designed our experiment to be as similar as possible to theirs.

We ran the experiments at the MaXLab, the experimental laboratory at the University of Magdeburg. For each neighborhood game, four sessions were conducted with 16 participants each. The participants were randomly matched, and they were told that the matching could change during the experiment—which did not, in fact, happen. We ran the games over 100 rounds. In each round the players could choose a different strategy against each neighbor. After each round the players were informed about the payoffs from each 2 x 2 game, i.e. about the total payoff from the last round and about the accumulated payoffs over all rounds.[6]

The payoffs were presented by "points" which were converted into Euro at an exchange rate of four payoff points equaling 1 Eurocent. An experimental session lasted about 1 to 1.5 hours and the average earnings of a participant were about 10 Euro. The 256 participants in our experiment were recruited using ORSEE software (Greiner, 2004) from a pool of mostly students from various faculties. To program our experiments we used the experimental software Z-tree (Fischbacher, 2007).

4. Experimental results

To compare our experimental results with the theoretical predictions of the five concepts, and also with the results of SC, in Table 1 we show the observed relative frequencies of playing *Up* (strategy "up" in the 2 x 2 games) and *Left* (strategy "left" in the 2 x 2 games), played in the two different network structures.

constant sum game							
		Session 1	Session 2	Session 3	Session 4	Average	Variance
Lattice	Up	0.032	0.053	0.061	0.045	0.048	0.000153
	Left	0.784	0.702	0.587	0.692	0.691	0.006529
Circle	Up	0.036	0.054	0.048	0.059	0.049	0.000098
	Left	0.779	0.683	0.756	0.641	0.715	0.004092
Non-constant sum game							
		Session 1	Session 2	Session 3	Session 4	Average	Variance
Lattice	Up	0.062	0.138	0.143	0.100	0.111	0.001425
	Left	0.677	0.764	0.795	0.741	0.744	0.002500
Circle	Up	0.115	0.107	0.110	0.051	0.096	0.000901
	Left	0.724	0.836	0.863	0.672	0.774	0.008223

Table 1: The relative frequencies of playing *Up* and *Left* in the baseline games

[6] We provided the same feedback as the players received in the experiment of Selten and Chmura (2008).

Key question 1: Do the participants (actually) play mixed strategies?

Unlike in other studies, the participants in our experiment could choose different strategies against each opponent in each period. Thus, the participants were able to mix their strategies within one round. As shown in Table 2, the participants did not use this opportunity very frequently.

Average No. of players choosing the same strategy against each neighbor over 100 rounds		
	Lattice	Circle
constant sum game	81.03%	85.69%
non-constant sum game	81.16%	86.22%

Table 2: Average No. of players choosing the same strategy against each neighbor over 100 rounds

The majority of players chose the same strategy for each neighbor. We do not find any difference in the strategy selection for the base games, but there are slight differences between the two different network structures. In terms of learning, or adjusting behavior, the results show that the frequency of choosing the same strategy against each neighbor increases over time (see Table 3).

Average No. of players choosing the same strategy against each neighbor over rounds 1 -50		
	Lattice	Circle
constant sum game	78.34%	83.06%
non-constant sum game	76.66%	83.09%
Average No. of players choosing the same strategy against each neighbor over rounds 51 - 100		
	Lattice	Circle
constant sum game	83.72%	88.31%
non-constant sum game	85.66%	89.34%

Table 3: Average No. of players choosing the same strategy in the first fifty and in the second fifty rounds

Result 1: Participants did not mix strategies within one round.

Key question 2: Does the structure of the network affect learning in games?

To find differences in learning between the Lattice network and the Circle network, we first analyze strategy selection in the baseline game. Based on the relative frequencies of playing *U* and *L* (see Table 1), we find no significant differences between the two network structures (sign test for any level of significance).

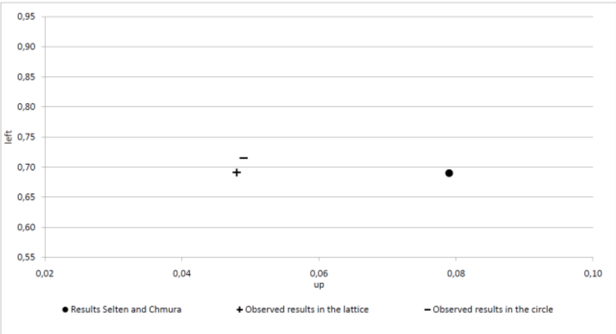
We compare our observed results with the experimental results of SC to analyze if there are at least differences between networks as compared to random matching environments with respect to learning behavior. Table 4 shows the observed results.

	Strategy choices – observed averages in:		
	Lattice network	Circle network	No network (Selten and Chmura)
	constant sum game		
Up	0.048	0.049	0.079
Left	0.691	0.751	0.690
	non-constant sum game		
Up	0.111	0.096	0.141
Left	0.744	0.774	0.564

Table 4: The observed averages of playing *Up* and *Left* in games

As one can see, our observed averages for both network structures are different from the results in the 2 x 2 games with only one randomly matched partner. The data in Figure 4 show that this is especially true for the non-constant sum game. These findings imply that learning in networks actually is different from learning in the 2 x 2 random matching environment used by SC.

Constant sum game



Non-constant sum game

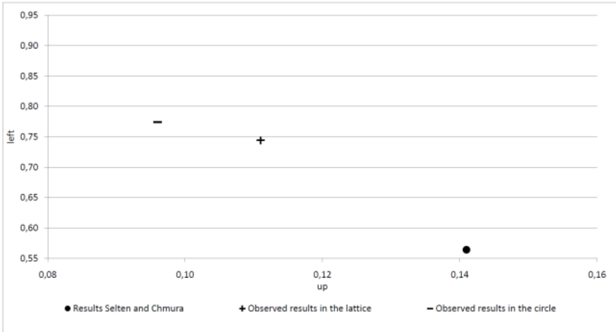


Figure 4: The observed averages of playing *U* and *L* in both games

Result 2: We found no significant differences in the participants' behavior in the two network structures.

Key question 3: Are the learning theories brought up by Selten and Chmura (2008) good predictors for the participants' behavior in network games?

As we want to answer the question about the predictive success of the five concepts of learning in the context of learning in networks, we first illustrate the theoretical predictions of the five concepts and the experimental results (Figure 5).

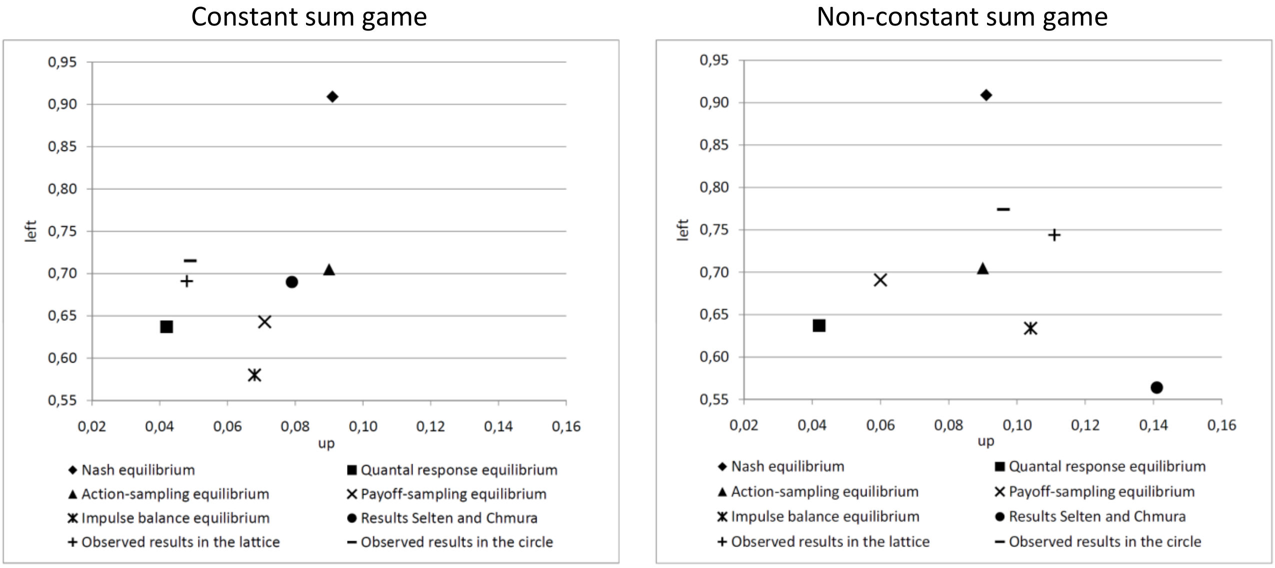


Figure 5: Theoretical predictions of the five stationary concepts and the experimental results

We use the corrected theoretical predictions of SC, according to BCG (2009). Table 5 shows the corresponding numerical predictions of the five stationary concepts.

	Nash	QRE	ASE	PSE	IBE	Observed average of Selten and Chmura	Our observed average in the Lattice	Our observed average in the Circle
	constant sum game							
U	0.091	0.042	0.090	0.071	0.068	0.079	0.048	0.049
L	0.909	0.637	0.705	0.643	0.580	0.690	0.691	0.751
	non-constant sum game							
U	0.091	0.042	0.090	0.060	0.104	0.141	0.111	0.096
L	0.909	0.637	0.705	0.691	0.634	0.564	0.744	0.774

Table 5: The theoretical predictions of the five stationary concepts and the observed relative frequencies of U and L strategies

To measure the predictive success of the stationary concepts, we analyze our data according to the method used by SC. The analysis is based on pairwise comparisons of the observed and the predicted relative frequencies. Table 6 shows the mean squared distances and the sampling variance for both base games played in the two network structures.

		Nash	QRE	ASE	PSE	IBE	Sampling variance
constant sum game	Lattice	0.054297	0.007988	0.006986	0.007880	0.017798	0.005011
	Circle	0.042619	0.009241	0.004899	0.008764	0.021652	0.003143
non-const. sum game	Lattice	0.030476	0.019173	0.004915	0.008355	0.015144	0.002943
	Circle	0.025158	0.028433	0.011603	0.016298	0.026441	0.006843

Table 6: Mean squared distances of the five stationary concepts

Based on the mean squared distances, it can be seen that there is an ordering of the concepts in terms of success: action-sampling equilibrium, payoff-sampling equilibrium, quantal response equilibrium, impulse balance equilibrium, and Nash equilibrium. For the non-constant sum game, the impulse balance equilibrium performs slightly better than the quantal response equilibrium.

Following the analyses in SC and in BCG, we test the results of all 16 independent observations together as well as separately for the constant and the non-constant sum game. Since the theoretical predictions are independent of the network structure, we test the results for both structures together. As in SC, we run the Wilcoxon matched-pairs signed rank test to compare the squared distances of the five concepts from the observed relative frequencies. In Table 7, we show the p-values in favor of the various concepts. As per the remarks in BCG, we also perform the Kolmogorov-Smirnov two-sample test to double-check the significance of the results (p-values are the numbers in brackets given in Table 7).

	QRE	ASE	PSE	IBE
Nash	**1% (5%)** *0.2% (2%)* n. s.	**0.001% (0.01%)** *0.02% (0.02%)* 5% (n. s.)	**0.05% (2%)** *0.1% (2%)* 10% (n. s.)	**5% (10%)** *2% (10%)* n. s.
QRE		**n. s.** *n. s.* n. s.	**n. s.** *n. s.* n. s.	**n. s.** *n. s.* n. s.
ASE			**n. s.** *n. s.* n. s.	**2% (10%)** *10% (10%)* 10% (n. s.)
PSE				**10% (n. s.)** *n. s.* n. s.

Table 7: p-values in favor of concepts, *above: 16 independent observations together*, *middle: constant sum game (eight independent observations), below: non-constant sum game (eight independent observations)*

When comparing all independent observations, it is obvious that all non-Nash concepts do significantly better than Nash[7]. This holds for the constant sum game but not for the non-constant sum game. Moreover, we found no significant clear order of predictive success among the four non-Nash concepts.

Result 3: There is no significant clear order of predictive success among the four non-Nash concepts.

5. Conclusion

In this paper, we analyzed learning in networks. Therefore, we added network structures to the 2 x 2 games taken from Selten and Chmura (2008). Starting with two different games (a constant and a non-constant sum game) we construct a neighborhood game with two different network structures (a lattice and a circle), where each player has four direct (local) neighbors. Unlike in other studies, the participants in our experiment could choose a different strategy against each neighbor.

As our study is related to that of SC, we compare our observed results to their experimental results, and to the theoretical predictions of five stationary concepts (Nash-equilibrium, quantal response

[7] That the non-Nash concepts outperform Nash is in line with the results of SC and BCG.

equilibrium, action-sampling equilibrium, payoff-sampling equilibrium, and impulse balance equilibrium).

Guided by our key questions, we first analyze our data to check if the participants really play mixed strategies. Because the majority of players choose the same strategy against each neighbor, we conclude that the participants did not mix their strategies in a round.

In a second step, we consider the influence of different network structures on the behavior of the participants. The differences between our results and the experimental results of SC provide evidence that learning in network differs from learning in 2 x 2 games with only one randomly matched partner. Unlike in other studies, we find no significant difference in the behavior of the participants in the two network structures we use. We find only slight differences in the number of participants choosing the same strategy against each neighbor.

Concerning our third key question, we show that none of the five stationary concepts provides an exact prediction for the participants' behavior in our experiment. We show that all non-Nash concepts outperform the Nash concept. The order of predictive success of the four non-Nash concepts is different for the constant sum and the non-constant sum game. It is perhaps notable that action-sampling equilibrium and payoff-sampling equilibrium strategies do better than impulse balance equilibrium when both games are combined. Aside from this, there is no clear ranking among these four concepts.

References

Bala, V., & Goyal, S. (2000). A Noncooperative Model of Network Formation. *Econometrica,* **68**(5), 1181-1229.

Berninghaus, S. K., Ehrhart, K.-M., & Keser, C. (2002). Conventions and Local Interaction Structures: Experimental Evidence. *Games and Economic Behavior,* **39**(2), 177-205.

Berninghaus, S. K., & Vogt, B. (2006). Network Formation in Symmetric 2 x 2 Games. *Homo Oeconomicus,* **23**(3/4), 421-466.

Brunner, C., Camerer, C. F., & Goeree, J. K. (2009). A Correction and Re-Examination of "Stationary Concepts for Experimental 2 x 2 Games", *Working Paper Series*. Zurich: Institute for Empirical Research in Economics, University of Zurich.

Cassar, A. (2007). Coordination and cooperation in local, random and small world networks: Experimental evidence. *Games and Economic Behavior,* **58**(2), 209-230.

Fischbacher, U. (2007). z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics,* **10**(2), 171-178.

Greiner, B. (2004). The Online Recruitment System ORSEE 2.0 - A Guide for the Organization of Experiments in Economics, *Working Paper Series of Economics*. Cologne: University of Cologne.

Jackson, M. O., & Watts, A. (2002). The Evolution of Social and Economic Networks. *Journal of Economic Theory,* **106**(2), 265-295.

Kirchkamp, O., & Nagel, R. (2007). Naive learning and cooperation in network experiments. *Games and Economic Behavior,* **58**(2), 269-292.

McKelvey, R. D., & Palfrey, T. R. (1995). Quantal Response Equilibria for Normal Form Games. *Games and Economic Behavior,* **10**(1), 6-38.

Osborne, M. J., & Rubinstein, A. (1998). Games with Procedurally Rational Players. *The American Economic Review,* **88**(4), 834-847.

Selten, R., Abbink, K., & Cox, R. (2005). Learning Direction Theory and the Winner's Curse. *Experimental Economics,* **8**(1), 5-20.

Selten, R., & Buchta, J. (1999). Experimental Sealed Bid First Price Auctions with Directly Observed Bid Functions. In D. Budescu, I. Erev & R. Zwick (Eds.), *Games and Human Behavior: Essays in the Honor of Amnon Rapoport* (pp. 79-104). Mahwah (NJ): Lawrence Erlbaum Associates.

Selten, R., & Chmura, T. (2008). Stationary Concepts for Experimental 2 x 2-Games. *The American Economic Review,* **98**(3), 938-966.

Selten, R., Chmura, T., & Goerg, S. J. (2010). Correction and Re-examination of Stationary Concepts for Experimental 2 x 2 Games: A Reply. *The American Economic Review,* **forthcoming**.

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