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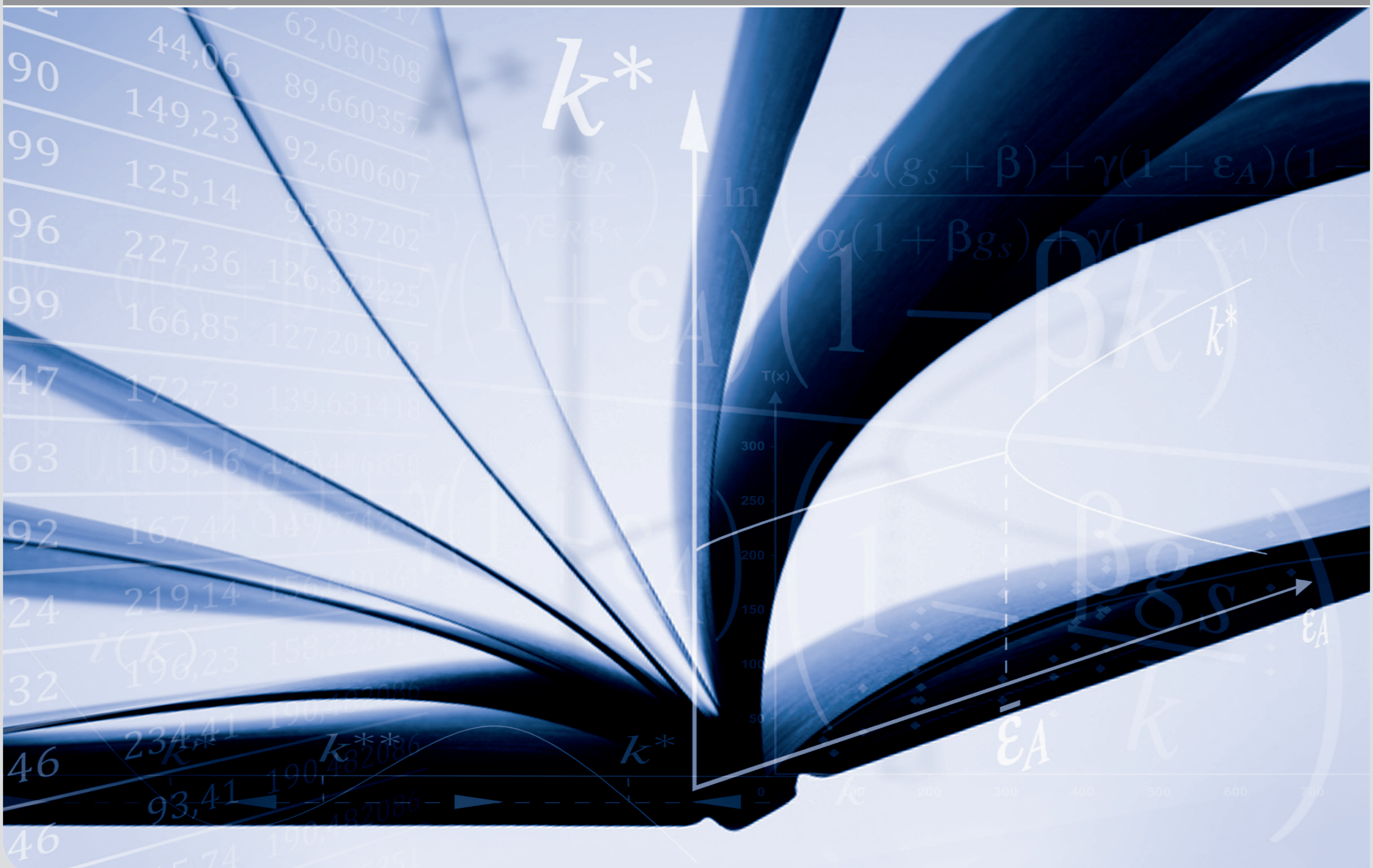

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Bayesian inference for hedge funds with stable distribution of returns

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Abstract

Recently, a body of academic literature has focused on the area of stable distributions and their application potential for improving our understanding of the risk of hedge funds. At the same time, research has sprung up that applies standard Bayesian methods to hedge fund evaluation. Little or no academic attention has been paid to the combination of these two topics. In this paper, we consider Bayesian inference for alpha-stable distributions with particular regard to hedge fund performance and risk assessment. After constructing Bayesian estimators for alpha-stable distributions in the context of an ARMA-GARCH time series model with stable innovations, we compare our risk evaluation and prediction results to the predictions of several competing conditional and unconditional models that are estimated in both the frequentist and Bayesian setting. We find that the conditional Bayesian model with stable innovations has superior risk prediction capabilities compared with other approaches and, in particular, produced better risk forecasts of the abnormally large losses that some hedge funds sustained in the months of September and October 2008.

Introduction

The financial crisis of 2008 had a devastating effect on the hedge fund industry and reshaped the way investors, risk personnel, and portfolio managers think about risk. According to Hedge Fund Research, total industry assets contracted by $461 billion in 2008 and nearly 1,000 hedge funds were liquidated. Hundreds of otherwise attractive, "safe" hedge funds found themselves unable to pay panicked investors in a timely fashion. Many were compelled to throw up gates, suspend redemptions, discontinue net asset value calculations, reorganize into illiquid side-pocket tranches, make payments-in-kind rather than cash or otherwise tamper with their ordinary terms and liquidity. For the investor, whether individual high net-worth, institution, or fund-of-funds, 2008 led

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to a sober reassessment of the tools and techniques for evaluating hedge fund risk. The entire spectrum of risk forecasting—from market and credit risk to liquidity analysis, operational due diligence and fraud mitigation to diversification—saw major upheavals and rethinking over the past year.

This chapter offers a new approach to forecasting the tail risk of hedge funds. While some researchers have studied the topic of Bayesian inference for stable distributions, no researchers to our knowledge have applied this analysis to the hedge fund industry. Furthermore, although numerous authors have applied Bayesian techniques to hedge fund performance, all have assumed normally distributed returns, ignoring the fat-tailed behavior described by the family of stable distributions. Finally, while researchers have touched on the topic of stable distributions and hedge funds, to our knowledge nobody has considered the analysis within a Bayesian framework.

One of the problems in evaluating the true tail risk of hedge funds is the lack of accurate performance data. Hedge funds often drop out of commercially available databases prior to revealing large losses. Many large, institutional-quality hedge funds choose not to report to public vendors whatsoever. To mitigate such concerns, we obtained proprietary data from a leading hedge fund-of-funds, whose database is several times larger than that of the public vendors. As such, we are able to investigate the complete and accurate performance histories of many active and dead hedge funds that are unavailable to any other researchers. We analyze the returns of a large collection of funds through the tumultuous market meltdown surrounding the Lehman bankruptcy in 2008, as well as the dramatic rebound of many survivors through December, 2009. Consequently, our conclusions regarding the fat-tailed behavior of hedge funds may be considerably more telling than the findings of most studies written prior to 2008-09.

The goal of the chapter is threefold. The first is illustrative: we discuss the stable density that, thanks to its heavy-tailedness and skewness, lends itself well to modeling hedge fund performance. Beginning with a frequentist overview, we proceed to describe a means of estimation of the parameters of the distribution in a Bayesian setting, in both unconditional and ARMA-GARCH contexts. This is followed by an example using simulated data. The second goal is to contrast the results of our risk evaluation methods with others in a broad, general context. We focus on the performance of the overall hedge fund industry as represented by a popular index, as well as the track record of an actual hedge fund with a long performance history. This comparison is more qualitative, and is meant to guide the typical hedge fund practitioner. The third goal also involves evaluating our models against alternatives, but in a more specific, rigorous vein. We assess how each model performs in forecasting value-at-risk (VaR) and conditional value-at-risk (CVaR) during the extreme market turmoil of 2008, with particular emphasis on the months of September and October of that year. We perform a battery of tests to determine whether the various approaches are properly specified, and also the degree of accuracy of each measure. Knowing now that 2008 was for most hedge funds the worst year ever experienced, we seek to determine whether our methods would have given superior forecasts well enough in advance for a risk or portfolio manager to have made meaningful preparation for the worst.

Literature Review

Until the mid 1990s, there was a dearth of research on Bayesian inference on stable distribution parameter estimation. Buckle (1995) is one of the earliest to implement a Markov Chain Monte Carlo (MCMC) algorithm (specifically, the Gibbs sampler), to make parametric and predictive Bayesian inference for stable distributions, and to generate Bayesian posterior samples from the parameters of a stable distribution with any prior distribution. The work by Buckle was followed by a slew of articles across disparate fields—computational statistics, finance/economics, signal processing, acoustics/speech, astronomy/astrophysics, pattern recognition, pharmacology, and genetics/biostatistics (gene expression profiling), among others. Qiou (1996) and Qiou and Ravishanker (1997, 1999) develop a sampling-based conditional Bayesian approach that simultaneously estimates the stable-law parameters and the parameters of a linear ARMA model, thus extending Buckle's approach to time series and multivariate sub-Gaussian ARMA problems. Ravishanker and Qiou (1998) further refine this research using Monte Carlo Expectation Maximization (MCEM). Godsill and Kuruoglu (1999) employ a hybrid rejection sampling and importance sampling scheme to implement MCMC and MCEM using a general framework involving scale mixtures of normals (SMiN). They claim their approach improves upon straightforward rejection sampling and Metropolis-Hastings approaches for symmetric stable models, and find use for this technique in the field of audio signal noise reduction. Tsionas (1999) likewise uses a SMiN representation limited to symmetric stable distributions with applications to econometric time series. Casarin (2004) generalized existing techniques to include Bayesian inference for mixtures of stable distributions, arguing that in some cases financial data exhibit not only heavy tails and skewness but also multimodality. Salas-Gonzalez, Kuruoglu, and Ruiz (2006a,b) employ a reversible-jump MCMC algorithm for parameter estimation of stable distributions involving impulsive, asymmetric, and multimodal data from the field of digital signal processing. Lombardi (2007) develops a random walk Metropolis sampler using a Fast Fourier Transform of the stable-law characteristic function to approximate the likelihood function, as explained in Rachev and Mittnik (2000).

Little has been written on stable distribution modeling of hedge fund returns. Olszewski (2005) fits a stable distribution to Hedge Fund Research (HFR) indices and runs simulations to generate returns; he then optimizes a fund-of-funds portfolio of these assets using a mean-CVaR objective function. The result is shown to be more efficient than other naive combinations. Literature on Bayesian inference for hedge fund returns is also scarce, and generally limited to the normal distribution case. Avramov, Kosowski, Naik and Teo (2007) and Kosowski, Naik, and Teo (2007) both use Bayesian approaches to determine that hedge funds do indeed produce alphas and exhibit return persistence. These studies, however, offer limited insight into the dynamics of hedge fund return distributions, and are merely extensions of research done on mutual funds. Agarwal, Bakshi, and Huij (2008) use a Bayesian approach to estimate alphas and factor sensitivities of hedge funds. Gibson and Wang (2009) improve upon Avramov et.al.'s Bayesian research by incorporating liquidity risk into the assessment of hedge fund returns.

Data Description

Hedge fund performance data was taken from a proprietary database of Alternative Investment Solutions (AIS), a large fund-of-funds group that is part of UBS's Alternative and Quantitative Investments platform. This database is several times larger than any commercial vendor platform and in fact is a superset of most all publicly available systems. As of this writing, the AIS database stores qualitative and quantitative information on over 20,000 programs and 45,000 share classes of these funds; the typical vendor lists only about 10,000 classes of 5,000 funds.[1] Moreover, the database contains the histories of thousands of funds long since liquidated; among these are entities dating back over four decades. Having access to a database used by the world's largest investor in hedge funds[2] allows for industry analysis that heretofore has been unattainable by academic researchers. For example, over 30% of the collection of funds in the AIS database are unknown to any vendor. This includes a substantial collection of the most desirable, successful, institutional-quality managers who typically do not report their results publicly. Such data have been obtained directly from primary sources including the hedge fund managers themselves, fund administrators, and prime brokers. A second benefit is that the data are thoroughly cleaned. By contrast, public providers often include numerous errors in their performance histories. Quite often, reports by different vendors on the performance of the same fund share class are inconsistent. A third benefit is that the track records of funds are far more complete than those provided to commercial vendors. AIS captures the complete track record of many funds that have ceased reporting to public databases. This includes both successful funds as well as those that suffered dramatic losses. As commercially available databases paint only a partial, and potentially inaccurate, picture of the hedge fund landscape, it becomes evident that quite possibly much academic research heretofore has biases more serious than previously thought.

The analysis is performed on a range of hedge fund strategies. Here too, we believe we make valuable improvements over previously published studies. Quite a number of hedge funds are misclassified into incorrect strategies by the hedge fund vendors, who in turn rely on the self-description coming from a manager or marketing agent. Extensive work by a team of practitioners has helped reclassify funds into more meaningful categories. The purpose of this classification is to determine whether any of the techniques we employ in this chapter prove more valuable for certain strategies over others.

To conduct the analysis, we initially draw a random sample of a bit over a hun-

[1] The convention used by many hedge fund vendors is to label each entity in their database collection a "fund." This is misleading as many of the vehicles reported by such suppliers are actually pari-passu tranches of larger programs; such tranches differ only by currency, fees, terms, leverage, hot issue eligibility, or domicile. The practice of AIS's database is to consider these items "share classes," of a common "fund" program. A "fund" is thus a unique trading approach or strategy taken by a manager; funds differ based on investment criteria, not accounting, financial or legal criteria. In this chapter, we select one share class per each fund as the representative class for statistical purposes. Typically this class is the one with the longest record, and whose fees and terms are most indicative of a USD-based day-one continuing investor.

[2] Institutional Investor lists UBS Alternative and Quantitative Investments's multi-manager platform as the world's largest hedge fund-of-funds with $32.286 billion in assets under management, as of January 1, 2010.

dred hedge funds with eight years of monthly in-sample performance data spanning January 2000 to December 2007. Using these 96 observations, we compute the expected VaR and CVaR for the following out-of-sample month, and compare it with actual performance. We repeat this process by rolling forward a month for a total of 24 out-of-sample months through December 2009. (Not all funds survived to the end.)

Unlike individual equities, equity indices, mutual funds, and other traded assets where researchers can take comfort in long histories of daily observations, hedge funds prove to be highly difficult vehicles to study due to the infrequency of their performance reporting (monthly) and lack of history (many funds live short lives, and many of today's managers started only recently). While our approach afforded us a relatively thorough sample size of numerous funds with nearly 100 observations, we note several data issues. First, funds chosen for this analysis have lengthy histories and are thus not indicative of the totality of all funds (alive and defunct). Second, the object of our analysis is to test critically different risk methodologies over one of the most tumultuous years in hedge fund history, namely 2008, and the dramatic recovery experienced by many managers who survived into 2009. However, the efficacy of VaR and CVaR models is better tested over longer business cycles. Third, we are limited in our assessment of the various risk models by the lack of out-of-sample observations. For a given fund, 24 data points are rather restrictive (compared with 250 daily observations in a year for a mutual fund or stock). Tests used in VaR backtesting may thus be lacking in power.

Methodology

A desirable characteristic of the return distribution is that it is flexible enough to accommodate varying degrees of tail thickness and asymmetry. Stable distributions are distributions with very flexible features, which nests as a special case the normal (Gaussian) distribution.[3] The criticism of stable distributions that has prevented them from becoming a mainstream distributional choice is the lack of a closed-form density function (with the exception of the three special cases mentioned below). While this criticism was valid at one time, the advances in computer power make their application increasingly accessible today. Rachev and Mittnik (2000) is a comprehensive source of information on alpha-stable distributions, their estimation, and numerous applications in finance. See also Stoyanov and Racheva-Iotova (2004) for a comparison of the efficiency of various numerical stable density approximation algorithms.

In this chapter, we employ two risk models based on the stable distribution in the Bayesian setting—an unconditional one and a conditional one—to model hedge fund returns. The conditional stable model has an ARMA(1,1)-GARCH(1,1) formulation and we propose its estimation as a two-stage process. First, we estimate an ARMA(1,1)-GARCH(1,1) process with Student's t-distributed innovations and then we fit an alpha-stable distribution to the standardized residuals, before computing the

[3] Stable distributions (both Gaussian and non-Gaussian) possess the property of stability (sums of stable random variables are themselves stable), which is clearly a desirable property for modeling returns. Moreover, a version of the Central Limit Theorem governs the asymptotic behavior of sums of stable random variables. Therefore, the financial modeling framework built around the normal distribution can be extended to the more general class of stable distributions.

expected risk measures. As part of our MCMC computational algorithm for estimation of the stable distribution, we employ the Fast Fourier Transform approach to stable density approximation of Rachev and Mittnik (2000).

Bayesian Estimation of the Parameters of the Alpha-Stable Distribution in the Unconditional Setting

Stable distributions are characterized by four parameters: tail parameter, α, skewness parameter, β, scale parameter, σ, and location parameter, μ, and are denoted by $S_\alpha(\beta, \sigma, \mu)$. We make the following prior assumptions for the parameters of the stable distribution in the unconditional setting: α and β have uniform distributions on the intervals $(1, 2)$ and $[-1, 1]$, respectively[4]; σ is modeled with a gamma distribution, and μ with a stable distribution.[5]

Since (the stable likelihood function and, therefore,) the log-posterior density is not available in closed form, we employ MCMC methods to simulate it. In particular, a modification of the Gibbs sampler, called the griddy Gibbs sampler, is used.[6] Developed by Ritter and Tanner (1992), the griddy Gibbs sampler is a combination of an ordinary Gibbs sampler and a numerical routine. Each parameter's conditional posterior density is evaluated numerically, on a grid of equally-spaced nodes spanning the effective support of the respective parameter. The supports of the stable parameters, α and β, are determined by the theoretical and empirical considerations that led to the choice of priors. The situation is less straightforward in the cases of σ and μ, as the definition of "effective support" changes, together with the sampler exploring the parameters' sampling space.[7] Since the number of grid nodes is fixed a priori, the larger the range of the grid selected, the more sparsely the grid spans that range. Then, it is possible that at a certain iteration of the sampler the value of the posterior density computed at the grid nodes is virtually zero, since most of the probability mass falls between two grid points (or outside of the grid range altogether). On the other hand, constructing a grid with a large number of grid nodes can make the numerical computations prohibitively expensive from a computational standpoint. The reason is that in a given iteration of the griddy Gibbs sampler, to compute the full conditional posterior density of a single parameter, the relatively computationally intensive FFT has to be performed (for each data point and each grid node) for a total of nm times, where n is the number of data points (hedge fund returns) and m is the number of grid nodes. We used 26-node grids for each of the four stable distribution parameters, while the length

[4]For the purposes of modeling returns, it is reasonable to assume that the tail parameter, α, takes values between 1 and 2. The characteristic function of the stable distribution is discontinuous for $\alpha = 1$. In order to avoid this problematic case and disregarding the trivial case of $\alpha = 2$ (corresponding to the normal distribution), the support of α is the open interval $(1, 2)$. The support of β is its theoretical support.

[5]The stable parameters are assumed to be independently distributed, although the independence assumption may be contended in the case of α and β. The skewness and the tail parameters are not independent: β becomes unidentified for $\alpha = 2$. The evidence in Rachev, Stoyanov, Biglova, and Fabozzi (2005) corroborates this. Specifying an appropriate joint distribution, though, is indeed a challenge. Lombardi (2004) provides a possible approach to the joint modeling of α and β.

[6]More information about computational Bayesian methods and MCMC can be found in the chapter by Robert and Marin in this volume.

[7]The only obvious constraint is that σ's support is the positive part of the real line.

of the sample used for calibration is 96 months.

If no prior intuition exists about what the likely parameter values are, a solution is to employ a variable instead of a fixed grid. Then, at each iteration of the sampling algorithm, one analyzes the distribution of the posterior mass of a parameter and adjusts spacing (equivalently, range) of the grid, so that the majority of the grid nodes falls into the interval of the greatest posterior mass (that is, into the effective parameter support). Extensive fine-tuning and sometimes multiple evaluations of the posterior density are required in the process.

Once the posterior density has been evaluated numerically, one needs to obtain the empirical cumulative distribution function (CDF) and draw from the parameter's posterior distribution using the CDF inversion method.

Bayesian Estimation of ARMA-GARCH Processes with Stable Disturbances

Our conditional modeling of hedge fund returns is based on the assumption that returns are linear functions of two components: a time-varying mean, μ_t, and an error term with a time-varying scale parameter, σ_t. Our model formulation is an ARMA(1,1)-GARCH(1,1) process, which specifies the conditional mean and variance equations as[8]

$$\begin{aligned} \mu_t &= \phi_0 + \phi_1 r_{t-1} + \phi_2 \epsilon_{t-1} \\ \sigma_t^2 &= \omega + \alpha\sigma_{t-1}^2 + \beta\epsilon_{t-1}^2, \end{aligned} \tag{1}$$

respectively, where $\epsilon_t = \sigma_t u_t$ is a zero-mean random noise and u_t are zero-mean, unit-scale independently and identically distributed (i.i.d.) random variables.

We develop our ARMA-GARCH process with stable innovations in a two-stage modeling procedure. In the first stage, we assume that the innovations, ϵ_t, are distributed with the Student's t distribution with ν degrees of freedom and scale σ_t and we estimate the ARMA-GARCH process in (1). This stage accounts for the volatility-clustering feature of hedge-fund returns and, to some degree, for their heavy-tailedness. Nevertheless, the standardized residuals of the ARMA-GARCH process still exhibit leptokurtosis and, moreover, are skewed. Therefore, in the second stage, we fit a stable distribution to the standardized residuals from stage 1, where the residuals are computed using the posterior means of the ARMA-GARCH process parameters.

In our empirical investigation, we consider the risk prediction capabilities of three conditional models, based on (1). The first one is an ARMA(1,1)-GARCH(1,1) model with Student's t innovations, estimated with the method of maximum likelihood. The second one is an ARMA(1,1)-GARCH(1,1) model with Student's t innovations, estimated in the Bayesian setting, using only step 1 of the procedure above. The third one is an ARMA(1,1)-GARCH(1,1) model with stable innovations, estimated in the Bayesian setting, using the complete two-stage procedure. In other words, the latter two models have a common Bayesian ARMA-GARCH estimation procedure. In the section on empirical results, we will label these models as Model 6, Model 7, and

[8] See any standard textbook on time series analysis for detailed definitions of conditional mean and volatility models, as well as their properties and stationarity, invertibility, and other constraints.

Model 8, respectively. Below, we outline in some more detail the two estimation stages of the ARMA-GARCH stable process (i.e., Model 8).

Stage 1: Bayesian Estimation of the ARMA(1,1)-GARCH(1,1) process with Student's t-distributed innovations

Uninformative prior distributions are asserted for the ARMA and GARCH parameters in (1). For the degrees-of-freedom parameter, ν, we assert an exponential prior distribution.[9] During the sampling process, we impose the stationarity, invertibility, and positivity constraints of the ARMA-GARCH process. The (covariance) stationarity constraint which, in a GARCH(1,1) model with Student's t-distributed innovations, takes the form $\alpha+\beta\nu/(\nu-2) < 1$,[10] is not enforced. Instead, one can observe whether that constraint is violated by examining the posterior distribution of the left-hand-side quantity.

The likelihood function for the ARMA(1,1)-GARCH(1,1) model with Student's t innovations is

$$L\big(\boldsymbol{\theta} \,|\, \boldsymbol{r}, \Im_0\big) \propto \prod_{t=1}^{T} \left[(\sigma^2_{t|t-1})^{-1} \left(1 + \frac{1}{\nu} \frac{\big(r_t - (\phi_0 + \phi_1 r_{t-1} + \phi_2 \epsilon_{t-1})\big)^2}{\sigma^2_{t|t-1}} \right)^{-\frac{\nu+1}{2}} \right], \tag{2}$$

where $\Im_0$ is the information set at the start of the process ($t = 0$). For simplicity, all information at $t = 0$ is assumed known; that is, ϵ_0 and σ_0^2 are known.[11]

The posterior density of the parameter vector, $\boldsymbol{\theta} = (\nu, \phi_0, \phi_1, \phi_2, \omega, \alpha, \beta)$, is then

$$p(\boldsymbol{\theta} \,|\, \boldsymbol{r}, \Im_0) \propto L\big(\boldsymbol{\theta} \,|\, \boldsymbol{r}, \Im_0\big)\pi(\nu) I_{\text{ARMA}} I_{\text{GARCH}}, \tag{3}$$

where I_{ARMA} and I_{GARCH} are the constraints on the ARMA and GARCH parameters, respectively.

When an estimation problem involving the Student's t distribution is cast in the Bayesian setting, it is convenient, from a computational point of view, to employ the scale-mixture of normals representation of the Student's t distribution, and we adopt that approach too. The conditional distribution of the additional T parameters, with which the parameter space is augmented, is simulated in the MCMC procedure, together with the posterior densities of the remaining parameters. For details on the forms of the likelihood function of the Student's t distribution, the scale-mixture representation of the Student's t distribution, and the forms of the posterior densities of

[9]Bauwens and Lubrano (1998) contend that if a diffuse prior on the interval $[0, \infty]$ is chosen for ν, then the posterior distribution is not proper (its right tail does not decay fast enough). One prior distribution option is a uniform distribution on the interval $[0, K]$, where K is some finite number. Our choice of prior follows Geweke (1993) who advocates the use of the exponential distribution, $\pi(\nu) = \lambda \exp(-\nu\lambda)$. Prior intuition can be used to determine exponential mean, $1/\lambda$.

[10]See, for example, Bauwens, Lubrano, and Richard (2000).

[11]It is also possible to treat ϵ_0 (and, therefore, σ_0^2) as an unknown parameter in the ARMA-GARCH process and simulate it together with the remaining parameters in the MCMC algorithm. See, for example, Chib and Greenberg (1994), among others.

the model parameters, see Chapters 10 and 11 of Rachev, Hsu, Bagasheva, and Fabozzi (2008).[12]

Two general approaches are available for the simulation of the posterior densities of ARMA and GARCH parameters: simulation parameter-by-parameter and simulation *en bloc*. We found that the parameter-by-parameter simulation results in a posterior sample of the ARMA parameters characterized by very high degree of autocorrelation and cross-correlation. Therefore, for both groups of parameters, we adopt the second approach and generate multivariate samples from the posterior distributions of the 3×1 vectors of ARMA and GARCH parameters. As proposal distributions, we use multivariate normal distributions with means given by the modes of the posterior kernels and covariance matrices given by the negative inverse Hessian matrices evaluated at the posterior modes.[13]

Stage 2: Fitting Stable Distribution to the Standardized ARMA(1,1)-GARCH(1,1) Residuals from Stage 1

Stage 2 is the step "upgrading" Model 7 (the conditional Student's t Bayesian model) to Model 8 (the conditional stable Bayesian model). Since we assume that the innovations, ϵ_t, of the ARMA(1,1)-GARCH(1,1) process are distributed with the Student's t distribution, the standardized residuals are given by

$$\hat{\epsilon}_t = \frac{r_t - \hat{\mu}_t}{\sqrt{\frac{\hat{\nu}}{\hat{\nu}-2}}\hat{\sigma}_{t|t-1}}, \tag{4}$$

where $\hat{\mu}_t$ is the vector of conditional means computed at the posterior means of the ARMA parameters, $\hat{\sigma}_{t|t-1}$ is the vector of conditional scales, computed at the posterior means of the GARCH parameters, and $\hat{\nu}$ is the posterior mean of the degrees-of-freedom parameter of the Student's t distribution. The term $\hat{\nu}/(\hat{\nu}-2)$ in the denominator is due to the variance of the Student's t distribution. We fit a stable distribution to the standardized residuals above, using the maximum-likelihood FFT approach of Rachev and Mittnik (2000).

Illustration with Simulated Data

We illustrate our Bayesian approaches to unconditional and conditional estimation by simulating samples of observations from an i.i.d. variable with the stable distribution and from the ARMA(1,1)-GARCH(1,1) with Student's t innovations (given in (1)) and then comparing the true parameters to their estimated counterparts.

Unconditional Stable Case

We generate a sample of 500 observations from a stable distribution and estimate its parameters using our MCMC procedure. The Markov chain is run for 10,000 itera-

[12] See also the contribution of Ardia and Hoogerheid in the current volume.

[13] The parameters of the proposal density are due to an asymptotic result from maximum-likelihood theory concerning the distribution of the maximum-likelihood estimator of the mean of the normal distribution. See Rachev, Hsu, Bagasheva, and Fabozzi (2008) for additional details.

tions. The first 2,000 of the simulations are discarded as burn-in. Table 1 presents the comparison among the Bayesian and frequentist estimates and the true parameters. The sample autocorrelations of the stable parameter simulations decay at a comfortable rate. Therefore, a procedure whereby the posterior parameter simulations are sampled periodically from the generated Markov chain is deemed unnecessary.

Conditional Student's t Case

We consider the ARMA(1,1)-GARCH(1,1) model in (1) with Student's t-distributed innovations. The generated sample consists of 3,000 observations. We estimate the conditional model using our MCMC approach, running the Markov chain for 10,000 iterations, and discarding as burn-in the first 2,000 of them. The comparison among the true and estimated parameters, together with the maximum-likelihood estimates, is shown in Table 2. The true parameter values fall into the 95% Bayesian credible intervals for all parameters, save for the degrees of freedom. Figure 1 presents the sample autocorrelations estimated using the after-burn-in posterior simulations of the AR, MA, GARCH, and ARCH parameters; all sample autocorrelation functions exhibit a fast decay.

Finally, we observe whether the GARCH(1,1) model stationarity constraint is violated by estimating the posterior probability of the persistence quantity, $\alpha+\beta\nu/(\nu-2)$, in the GARCH (covariance) stationarity constraint. The posterior mean of this persistence quantity is 0.8334 which is below 1 and signifies a conditional process with finite variance (for comparison, the true value of the quantity is 0.95). The histogram of its posterior draws is seen in Figure 2. The greater part of the posterior mass is indeed below 1, as expected.

Value-at-Risk and Conditional Value-at-Risk Prediction

In our empirical investigation, we estimate VaR and CVaR for a number of risk-model formulations, the first two of which are very basic standard methodologies, still used by many banks; we include them for benchmarking purposes. We label these formulations as Model 1 through Model 8, as follows:[14]

Model 1: Unconditional (i.i.d.) normal model, estimated in a frequentist (maximum-likelihood) setting

Model 2: Historical VaR/CVaR methodology

Model 3: Unconditional (i.i.d.) stable model, estimated in a frequentist setting

Model 4: Unconditional (i.i.d.) stable model, estimated in a Bayesian setting

Model 5: Unconditional (i.i.d.) Student's t model, estimated in a frequentist setting

[14] For the definitions and properties of various risk measures, in particular VaR and CVaR, as well as their applications in risk and portfolio management, see, for example, Rachev, Stoyanov, and Fabozzi (2008).

Model 6: Conditional (ARMA(1,1)-GARCH(1,1)) Student's t model, estimated in a frequentist setting

Model 7: Conditional (ARMA(1,1)-GARCH(1,1)) Student's t model, estimated in a Bayesian setting

Model 8: Conditional (ARMA(1,1)-GARCH(1,1)) stable model, estimated in a Bayesian setting.

Recall that Models 7 and 8 have a common Bayesian ARMA-GARCH estimation step (described as Stage 1 in the section on methodology), while Model 8 is obtained via an additional step of fitting the stable distribution to the standardized residuals from the common step (described as Stage 2 in the section on methodology).

In all models, VaR and CVaR estimation is based on the linear-form decomposition of returns, $R_t = \mu_t + \sigma_t u_t$, where u_t is a noise term with the respective distribution (normal—in Model 1, stable—in Models 3, 4, and 8, and Student's t—in Models 6 and 7). Based on the available information up to time t, and using their translation invariance and positive homogeneity properties, the VaR and CVaR estimates are expressed, respectively, as

$$\widehat{VaR}_{\kappa,t} = \hat{\sigma}_t VaR_\kappa(u) - \hat{\mu}_t \quad (5)$$

$$\widehat{CVaR}_{\kappa,t} = \hat{\sigma}_t C_\kappa(u) - \hat{\mu}_t, \quad (6)$$

where $VaR_\kappa(u)$ is the value-at-risk (the κ quantile) of the innovation's (u_t's) distribution, and $C_\kappa(u)$ is a constant which depends only on the tail probability, κ.[15] (In our discussion below, we omit the "hats" on VaR and CVaR for notational simplicity.) In the unconditional model approaches (Models 1 through 5), $\hat{\mu}_t$ and $\hat{\sigma}_t$ are constants represented by the sample estimates and hedge fund returns are assumed independent and identically distributed with the respective distributions. In the conditional modeling approaches (Models 6 through 8), $\hat{\mu}_t$ and $\hat{\sigma}_t$ are the forecasts of the conditional mean and conditional scale from the ARMA(1,1)-GARCH(1,1) model in (1).[16]

We now outline the explicit and semi-explicit expressions used to compute CVaR for the normal, Student's t, and stable distributions.

The Normal Distribution

For a normal distribution with standard deviation σ and expected value μ, the CVaR is expressed as

$$CVaR_\kappa(R) = \frac{\sigma}{\kappa\sqrt{2\pi}} \exp\left(-\frac{(VaR_\kappa(Z))^2}{2}\right) - \mu, \quad (7)$$

where Z is a standard normal random variable.

[15] The VaR and CVaR estimates above are computed at a time horizon of one month in our empirical investigation.

[16] For more details on VaR/CVaR prediction using a time-series model, see, for example, Tsay (2005).

The location-scale Student's t Distribution

In the case of a location-scale Student's t distribution with degrees of freedom ν, scale σ, and location μ, CVaR is computed from the following explicit expression:[17]

$$CVaR_\kappa(R) = \begin{cases} \frac{\sigma\Gamma(\frac{\nu+1}{2})}{\kappa\Gamma(\frac{\nu}{2})} \frac{\sqrt{\nu-2}}{(\nu-1)\sqrt{\pi}} \left(1 + \frac{(t_\nu^{-1}(\kappa))^2}{\nu-2}\right)^{-\frac{\nu-1}{2}} - \mu & ,\nu > 1 \\ \infty & ,\nu = 1, \end{cases} \tag{8}$$

where $\Gamma(x)$ is the gamma function and $t_\nu(\kappa)$ is the κ-quantile of a standardized (zero mean and variance equal to 1) Student's t-distributed random variable with ν degrees of freedom.[18]

The Stable Distribution

Stoyanov et.al. (2006) derived the semi-analytical expression for the CVaR for stable distributions. The $CVaR_\kappa$ is represented as

$$CVaR_\kappa(R) = \sigma A_{\kappa,\alpha,\beta} - \mu. \tag{9}$$

The term $A_{\kappa,\alpha,\beta}$ is given by

$$A_{\kappa,\alpha,\beta} = \frac{\alpha}{1-\alpha} \frac{|VaR_\kappa(R)|}{\pi\kappa} \int_{-\bar{\theta}_0}^{\pi/2} g(\theta) \exp\left(-|VaR_\kappa(R)|^{\frac{\alpha}{\alpha-1}} \upsilon(\theta)\right) \mathrm{d}\theta,$$

where

$$\begin{aligned} g(\theta) &= \frac{\sin(\alpha(\bar{\theta}_0 + \theta) - 2\theta)}{\sin(\alpha(\bar{\theta}_0 + \theta))} - \frac{\alpha\cos^2\theta}{\sin^2(\alpha(\bar{\theta}_0 + \theta))}, \\ \upsilon(\theta) &= (cos\alpha\bar{\theta}_0)^{\frac{1}{\alpha-1}} \left(\frac{\cos\theta}{\sin(\alpha(\bar{\theta}_0 + \theta))}\right)^{\frac{\alpha}{\alpha-1}} \frac{\cos(\alpha\bar{\theta}_0 + (\alpha-1)\theta)}{\cos\theta}, \end{aligned}$$

and $\bar{\theta}_0 = \frac{1}{\alpha}\arctan(\bar{\beta}\tan\frac{\pi\alpha}{2})$, $\bar{\beta} = -\mathrm{sign}(VaR_\kappa(R))\beta$, $VaR_\kappa(R)$ is the VaR of the stable distribution at tail probability κ, and β is the stable skewness parameter. The parameters of the stable distribution are estimated either in the frequentist or the Bayesian setting.

Backtesting VaR and CVaR

We backtest the risk models using the Kupiec (1995) frequency of failures test and the Christoffersen (1998) test of independence of the VaR violations. For backtesting CVaR, we use a loss function-based procedure.[19] Our general backtesting process

[17] See, for example, Alexander and Sheedy (2008).

[18] For $\nu = 1$, CVaR explodes because the Student's t distribution with one degree of freedom—known as the Cauchy distribution—has an infinite expectation. In this case, one can use the median of the loss distribution, when the loss exceeds $VaR_\kappa(R)$, as a robust alternative to CVaR. See Rachev, Stoyanov, and Fabozzi (2008) for more details.

[19] Backtesting CVaR is a challenging task. See Rachev, Stoyanov, and Fabozzi (2008) for a discussion.

consists of repeatedly estimating VaR and CVaR based on a moving estimation window and comparing the predicted risk values to the out-of-sample realization of returns one-step-ahead. That is, the sequences of VaR and CVaR estimates are based on the updated (revised) parameter estimates using the latest estimation window. An exceedance of the VaR occurs when the realized loss is greater than the predicted VaR for the one-step-ahead horizon. Next, we describe the CVaR backtesting procedure.

CVaR Backtesting Procedure

Our CVaR backtesting procedure relies on a loss function, developed in the spirit of Blanco and Ihle (1999)'s loss function for ranking models based on their VaR predictive capacity.[20] Denote the loss at time t by L_t. The loss function is defined as

$$LF_t = \begin{cases} \frac{L_t - CVaR_{\kappa,t}}{L_t}, & \text{if } L_t > VaR_{\kappa,t} \\ 0, & \text{if } L_t \leq VaR_{\kappa,t} \end{cases}. \tag{10}$$

Then, the statistic

$$S = \sqrt{\frac{1}{T}\sum_{t=1}^{T} LF_t^2}, \tag{11}$$

where T is the sampling horizon, provides a summary metric for the average distance of the forecast CVaR from the realized loss, in the case of VaR exceedance. In our empirical analysis, we compute this statistic for each model and each hedge fund in our sample universe.

Empirical Analysis

Our empirical analysis consists of four parts. First, we analyze and compare the models' risk forecasts using hedge fund index data. Second, we focus on the performance of a particular convertible arbitrage hedge fund that experienced a large loss in 2008 and staged a strong recovery in the following year. Third, we perform a general comparison among models' VaR and CVaR predictions across six hedge fund strategies most deeply impacted by the recent financial crisis: merger arbitrage, convertible bond (CB) arbitrage, directional credit (distressed debt and high-yield), long/short (LS) credit, fixed income (FI) arbitrage, and mortgage-backed security (MBS) arbitrage. Finally, we focus our attention on the momentous months of September and October 2008, with the aim of comparative evaluation of models across the hedge fund strategies. In all four investigations, we use eight years of monthly data in-sample (starting in January 1990 for the hedge fund index data and in January 2000 for the individual hedge fund data) and compare methodologies on a rolling month-by-month out-of-sample basis (spanning 12 years for the hedge fund index and two years for the remaining data). Our sample universe consists of 27 funds in the merger arbitrage strategy, 17 funds in

[20] Dowd (2008) provides an overview of backtesting market risk models. For a suggestion on a more rigorous approach to CVaR backtesting, see Rachev, Stoyanov, and Fabozzi (2008).

the CB arbitrage strategy, 40 funds in the directional credit strategy, 17 funds in the LS credit strategy, 16 funds in the FI arbitrage strategy, and 10 funds in the MBS arbitrage strategy.

Comparisons of Risk Forecasts on Hedge Fund Index Data

The HFRI Fund Weighted Composite Index is widely used as an indicator of the performance of the overall hedge fund industry. Comprised of over 2,000 funds listed in the internal HFR database, the index is an equally weighted average of monthly returns net of fees starting in January 1990. As such, the index records hedge fund trends over a lengthy business cycle of busts and booms. This evaluation includes a number of major crisis periods ranging from the Long Term Capital/Russian default of August 1998, the terrorist attack of September 2001, the credit crunch of Summer 2002, the credit correlation crisis of Spring 2005, the market meltdown of May 2006, the quant debacle of August 2007, the subprime jitters of November 2007, the market corrections of January and March 2008, and, ultimately, the Lehman bankruptcy/market collapse of September-October 2008. In all, there were 14 monthly events in 12 years (just about one in 10 times) where performance, as measured by total return, was less than -2%, yet the overall average return during this period was a positive 0.66%.

As seen in Figure 3, the -8.7% drop in August 1998 and the back-to-back losses of 6.13% and 6.84% in September and October 2008 sent hedge fund investors into a state of panic. We observe that traditional risk models—naive, historical, and Student's t—did poor jobs not only in forecasting the severity of these downturns but also in adjusting properly afterwards. For example, after the massive LTCM shock, simple and historical models did not adjust their risk prediction much at all, and continued to forecast inadequately a VaR at the 95% level of around -2%. Worse, CVaR forecasts for August, 1998 were in the -2% range for the non-Bayesian approaches at the 95% level and only -3% at the 99% level. These results are very difficult to accept for risk managers, portfolio decision-makers, and investors, all of whom demand models that make meaningful forecasts when crises hit, not merely during normal market conditions!

Our first observation is that the stable models tended to do a much better job anticipating these large tail events than the non-stable approaches and that the Bayesian methodologies proved superior to the frequentist forecasts. The conditional stable Bayesian model anticipated the losses of August 1998 and September 1998 with predictions of -6.50% and -5.77% of the 99% VaR, while the CVaR forecasts were quite close at -12.04% and -6.58%, respectively. However, our second observation is that the conditional models—and the conditional stable Bayesian solution, in particular—substantially overshot (in terms of both VaR and CVaR) the actual performance in months subsequent to the large market dislocations and the CVaR forecasts of these models took considerable time to re-adjust to more rational levels. While the Bayesian and stable methods were suggesting the potential for worsening conditions, markets rebounded and hedge funds shook off these isolated losses.[21] Of all the CVaR risk esti-

[21] That the conditional stable Bayesian model suggested considerable losses subsequent to the major market events of August 1998 and September/October 2008 was not lost on many individual hedge funds which did in fact lose almost all their value and liquidate. The analysis herein involves an industry index which has some biases in that it often fails to include the performance of funds that are terminating. Since the index will

mates, the i.i.d. stable Bayesian, the conditional Student's t frequentist, and conditional Student's t Bayesian models seemingly did best over the two-month consecutive meltdown of September-October 2008, neither dramatically under-, nor over-forecasting losses.

How well did the different risk methods perform in the overall timeframe? The Kupiec test at the 95% level suggests a non-rejection range of between 3 and 12 exceedances inclusive; the i.i.d. stable Bayesian and conditional stable Bayesian approaches both had only two exceedances and could be considered to be misspecified over the long-term period (though the null hypothesis could not be rejected at the 99% level for any test). The Christoffersen test was inconclusive for all of the models. As for the comparison based on deviation size, the conditional stable Bayesian approach did the best job of explaining performance conditional upon there being an exceedance of the VaR level.

Comparison of Risk Forecasts on An Individual Hedge Fund

Rather than looking at hedge fund index data, which can in many ways be artificial and misleading, we contrast the different risk models using actual hedge fund performance. We examine an interesting (CB) arbitrage fund that has been around for over 20 years, experiencing not only the market shocks described in the previous section but also the idiosyncratic busts that affected the convertibles market and the CB arbitrage strategy, especially in the period 2004-2005. At its peak, this fund commanded nearly $400 million in assets. Since its inception, the fund has produced a solid average monthly return of 0.65% with low volatility and a Sharpe ratio exceeding 1. It suffered a drawdown of 15% from May 2008 to July 2009 (in itself a remarkable feat as many other funds lost twice that) including a single worst monthly loss of around 9% in October 2008. The fund got caught in the Mandalay Bay dividend crunch of 2003 and suffered losses in 2004 due to rising yields, lackluster primary market issuance, and low implied volatility. In 2005, the fund like many others was caught by surprise at General Motors' profit warning; the result was a loss of over 3% in April, 2005. Amidst massive investor redemptions, the fund recovered and turned strong profits in 2006 and 2009.

Over the period 2008-2009, the Kupiec and Christoffersen tests could not be rejected as the two exceedances of 2008 were not unexpected. As with the HFRI analysis in the previous section, we find that the conditional stable Bayesian model was the only model that recognized the severity of the large September/October 2008 losses. The 95% VaR forecasts were -1.5% and -8.2%, and the 95% CVaR forecasts were -3.9% and -18.4% for the months of September and October 2008, respectively, while realized losses were 3.7% and 9.4%, respectively. As also reported above, the non-Bayesian VaR models failed to budge much after the 2008 meltdown, though the Bayesian methodologies (and the conditional ones, in particular) greatly overestimated the losses in later months, when in fact the CB aribtrage manager was staging a recovery. The conditional models' VaR forecasts returned to reasonable levels as the hedge fund showed only positive gains in 2009; however, these models continued to exhibit

continue indefinitely as an average of surviving and existing funds, it is no surprise that some risk estimators will tend to overshoot after a major market event.

uncharacteristically high CVaR estimates after the markets calmed.

The lesson for risk managers from the findings reported in this and the previous sections may be that the conditional Bayesian models seem to give the best advance warning of the tail risk in hedge funds, but should be treated carefully after a large market dislocation—hedge funds tend to either liquidate or bounce back strongly.

Comparison of Risk Forecasts on a Panel of Many Funds Across Different Strategies

Performance According to Kupiec and Christoffersen Tests

The Kupiec test for unconditional coverage and the Christoffersen joint test for unconditional and independence are performed for each hedge fund within our sample universe, on the basis of the 24 out-of-sample forecasts of VaR at the 95% and 99% levels. We report our results summarized along the hedge fund strategy dimension and the model dimension. The conditional stable Bayesian model seemingly performs best among the eight models in terms of overall quality of its VaR prediction—the Kupiec and Christoffersen tests are rejected for only one hedge fund in each of the directional credit, LS credit for the $\mathrm{VaR}_{0.05}$ forecasts; while in the FI arbitrage strategy case, they are rejected for one and two hedge funds, respectively. Overall, the three conditional models—stable Bayesian, Student's t Bayesian, and Student's t frequentist—demonstrate superior performance, with the Student's t Bayesian having an upper edge over the frequentist one. The percentages of hedge funds for which the Kupiec and Christoffersen tests are rejected, summarized by model and strategy, are given in Table 3.

CVaR Performance According to the Loss Function Metric

As expected, based on the results from the Kupiec and Christoffersen tests, the conditional stable Bayesian model outperforms the rest in terms of accuracy of its VaR forecasts, as seen in Tables 4 through 6. It has the lowest incidence of VaR exceedances among the eight models for the overall out-of-sample period January 2008-December 2009. It is interesting to note that in the case of the CB arbitrage strategy, violations of the predicted VaR at 95% and 99% are much more numerous across all models, compared with other strategies (Table 4). CB arbitrage suffered more than other approaches not only from market losses but also as a result of larger-than-expected investor redemptions that forced funds to sell at the most inopportune times to raise cash. By contrast, most models do an adequate job in predicting VaR at 95% level in the case of the merger arbitrage strategy, while the conditional stable Bayesian approach fared best overall.

When the predicted VaR levels are violated, analysis of the distance between the realized loss and the predicted CVaR could give an indication of whether a risk manager would have had an adequate warning signal as to the potential average loss. The eight models are ranked across strategies on the basis of the achieved distance, with higher rank signifying smaller difference between realized loss and predicted CVaR, conditional on a VaR violation. Comparison across the strategies and models unsurprisingly again singles out the three conditional models. The conditional stable Bayesian model, however, is invariably ranked first for the greatest proportion of hedge funds; for example, for 33.3%, 76.5%, 40%, 47.1%, 37.5%, and 40% of the funds at the 95% VaR/CVaR level for the merger arbitrage, CB arbitrage, directional credit, LS credit strategies, FI arbitrage, and MBS arbitrage, respectively, when its predicted 95% VaR is exceeded. The conditional Student's t Bayesian model demonstrates an overall marginal performance advantage over the frequentist one, based on this distance metric.

Risk Performance During the Months of September and October 2008

As mentioned in previous sections, risk forecast comparison for the months of September and October 2008 is informative in view of the unusually large losses sustained by many hedge funds during this period. In order to gain more insight into the performance of the models, in this section we analyze two metrics: (1) the distance between realized loss and CVaR, conditional on VaR exceedance (as above) and (2) the distance between realized loss and VaR, conditional on VaR exceedance. We perform this analysis because it is important to distinguish among models whose VaR forecast undershoots the realized loss by a large amount and those whose VaR predictions are violated only marginally.

Again, one can notice that the conditional stable Bayesian model comes closer than the other models in terms of accuracy of VaR predictions with up to 44% and 96% of hedge funds having accurate VaR predictions at the 95% level in September and October 2008, respectively (for the merger arbitrage strategy).[22] When the 95% VaR is exceeded, the conditional stable Bayesian models' risk forecasts are closest to the realized losses.[23] The performance of the conditional Student's t Bayesian and frequentist models is comparable, as can be seen in Tables 7, 8, 10, and 11.

The difference in models' risk performance for the CB arbitrage strategy in September 2008 compared to October 2008 is worth noting. For hedge funds in this strategy, October was by far the worse of the two months, with 15 out of the 17 funds experiencing sometimes more than twice as severe loss in October compared to September. This unprecedented tail event was not seen in other strategies. The severity of the September loss, however, led to a large forecasted risk for October by the three conditional models, with the stable Bayesian model predicting accurate VaR at the 95% level for more than 60% of the hedge funds.

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[22] Due to space considerations, we discuss and present only the results concerning the 95% VaR and CVaR forecasts. The 99% VaR/CVaR results for the months of September and October 2008 per model and strategy are available from the authors upon request.

[23] Notice that in certain instances, for example, the CB arbitrage October 2008 results in Table 10, the directional credit October 2008 results in Table 11, or the FI arbitrage October 2008 results in Table 12, Model 7 (conditional Student's t Bayesian model) has been ranked first for a greater proportion of hedge funds in the respective strategy than Model 8 (conditional stable Bayesian model). These results are produced by virtue of the fact that there are more funds for which Model 7's VaR is exceeded than Model 8's and should not mislead into interpreting Model 7 as superior to Model 8. When rankings are viewed in conjunction with the results on VaR exceedances at the bottom of each table, it is evident that Model 8's risk performance is better.

Conclusion

In this chapter, we apply the Bayesian methodology to analyzing hedge fund risk with a conditional time series model with stable innovations. We compare that model's out-of-sample risk forecasting performance to that of seven competing models estimated in the frequentist and Bayesian setting. Our analysis shows an advantage of the conditional stable Bayesian model in predicting both VaR and CVaR in general, and in particular, as far as the crisis months of September and October, 2008, are concerned. The conditional Student's t models (estimated in the Bayesian and frequentist setting) perform better among the remaining models, with the Bayesian variety seemingly having a slight edge. Among the six hedge fund strategies we investigate, the convertible bond arbitrage strategy seems to pose the biggest challenge for our models, with fewest instances where the models' risk forecasts "caught" losses in September 2008. Even then, though, the conditional stable Bayesian model's forecasts are closest to the realized losses. Risk forecasts for October 2008, however, adjust and are adequate for a greater proportion of hedge funds in that strategy (with the conditional stable Bayesian model predicting risk best).

Given that in periods of market rebound the three conditional models are slow to "catch on" suggests that there may be no "one model fits all" solution and risk managers may wish to employ different models in different market regimes. Additionally, even though our empirical analysis suggests that in periods of market distress the conditional stable Bayesian model offers the most adequate risk predictions across all hedge fund strategies, a longer backtesting period may reveal additional insights in terms of across-strategy and across-model comparisons.

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Tables and Figures

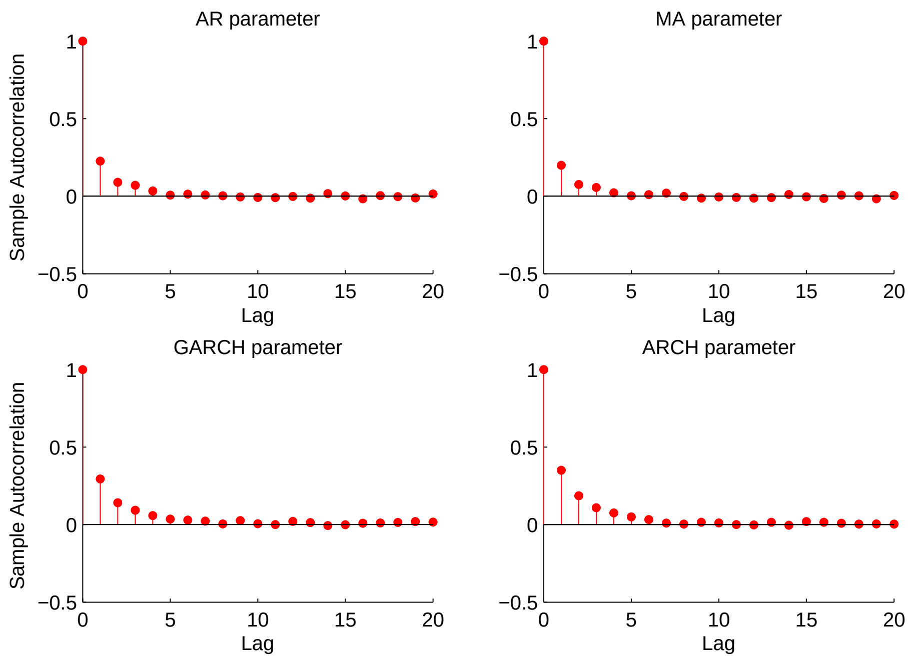

Figure 1: Sample autocorrelations of posterior draws from the distributions of the AR, MA, GARCH, and ARCH parameters

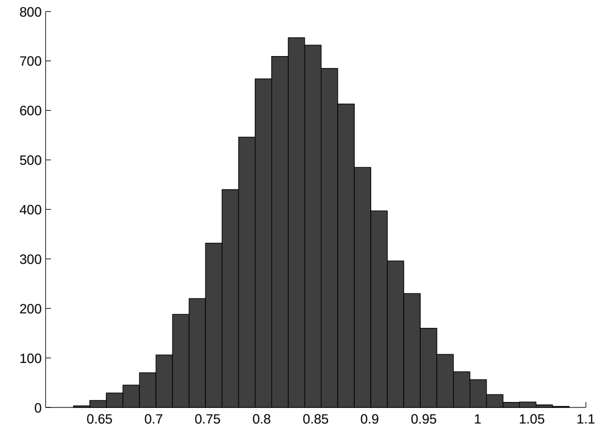

Figure 2: Histogram of draws from the posterior distribution of the GARCH(1,1) stationarity quantity

Parameter	Simulated Parameter Value	Bayesian Posterior Mean	Maximum Likelihood Estimate
α	1.7	1.2464 $(1.1300, 1.3662)$	1.7471
β	0.2	0.2733 $(0.1219, 0.4183)$	-0.1626
σ	0.3	0.1653 $(0.1497, 0.1813)$	0.2786
μ	0.05	-0.0210 $(-0.0530, 0.0110)$	0.0429

Table 1: Simulation Results: Stable i.i.d. Bayesian Model Case. The numbers in brackets are the 95% Bayesian coverage intervals based on 8,000 simulations (after-burn-in) of the MCMC Stable i.i.d. model estimation procedure.

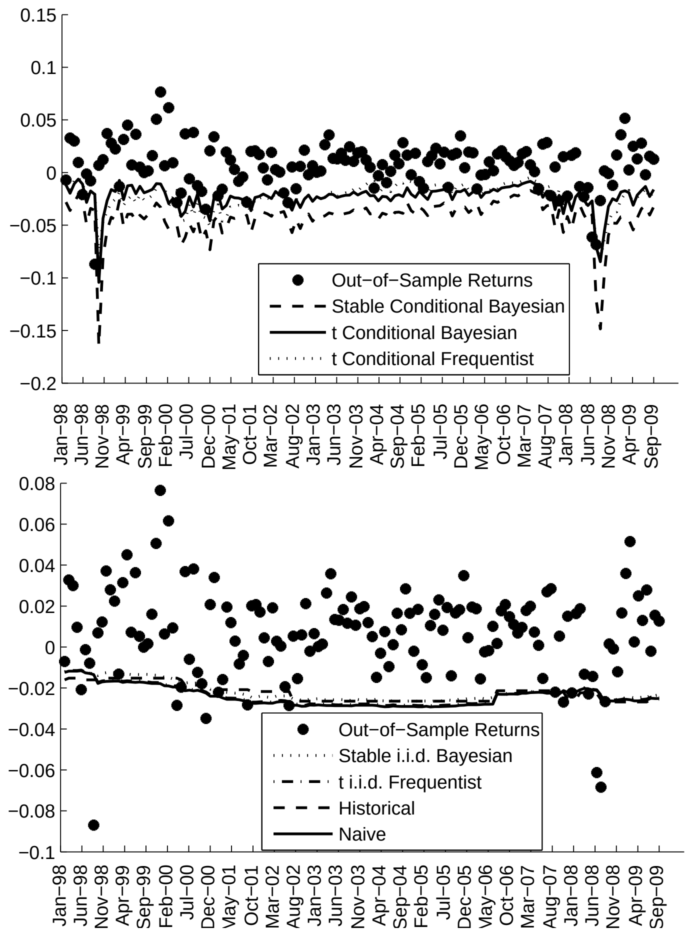

Figure 3: 95% VaR Predictions and Out-of-Sample HFRI Returns

Parameter	Simulated Parameter Value	Bayesian Posterior Mean	Maximum Likelihood Estimate
ϕ_0	0.01	0.0109 $(0.0087, 0.0133)$	0.0105
ϕ_1	0.65	0.5987 $(0.5175, 0.6716)$	0.6112
ϕ_2	-0.35	-0.3248 $(-0.4174, -0.2286)$	-0.3405
ω	0.001	0.0010 $(0.0008, 0.0013)$	0.0011
α	0.55	0.4477 $(0.3336, 0.5593)$	0.4685
β	0.2	0.2389 $(0.1758, 0.3089)$	0.2518
ν	4	5.2546 $(4.45, 6.15)$	4.1797

Table 2: Simulation Results: ARMA-GARCH Bayesian Model Case. The numbers in brackets are the 95% Bayesian coverage intervals based on 8,000 simulations (after-burn-in) of the MCMC conditional model estimation procedure.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Kupiec Test				**Merger Arbitrage**				
	25.9%	29.6%	29.6%	25.9%	29.6%	11.1%	7.4%	0.0%
	11.1%	0.0%	0.0%	11.1%	0.0%	0.0%	0.0%	0.0%
				Convertible Bond Arbitrage				
	41.2%	52.9%	52.9%	41.2%	52.9%	23.5%	17.6%	0.0%
	5.9%	0.0%	5.9%	0.0%	0.0%	0.0%	0.0%	0.0%
				Directional Credit				
	45.0%	50.0%	55.0%	50.0%	52.5%	15.0%	12.5%	0.0%
	12.5%	0.0%	2.5%	15.0%	0.0%	0.0%	0.0%	0.0%
				Long/Short Credit				
	47.1%	41.2%	52.9%	58.8%	58.8%	11.8%	5.9%	0.0%
	11.8%	11.8%	5.9%	17.6%	5.9%	0.0%	0.0%	0.0%
				Fixed Income Arbitrage				
	18.8%	37.5%	37.5%	25.0%	25.0%	0.0%	6.3%	6.3%
	6.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
				MBS Arbitrage				
	20.0%	30.0%	30.0%	40.0%	30.0%	20.0%	10.0%	0.0%
	10.0%	0.0%	10.0%	20.0%	0.0%	0.0%	0.0%	0.0%
Christoffersen Test				**Merger Arbitrage**				
	63.0%	48.1%	55.6%	63.0%	66.7%	29.6%	11.1%	0.0%
	37.0%	7.4%	3.7%	33.3%	0.0%	0.0%	0.0%	0.0%
				Convertible Bond Arbitrage				
	41.2%	70.6%	64.7%	41.2%	64.7%	29.4%	23.5%	0.0%
	17.6%	0.0%	11.8%	5.9%	11.8%	0.0%	0.0%	0.0%
				Directional Credit				
	75.0%	80.0%	87.5%	77.5%	87.5%	47.5%	47.5%	2.5%
	60.0%	12.5%	30.0%	52.5%	12.5%	15.0%	5.0%	0.0%
				Long/Short Credit				
	58.8%	64.7%	64.7%	64.7%	70.6%	41.2%	29.4%	5.9%
	47.1%	17.6%	29.4%	47.1%	23.5%	11.8%	5.9%	0.0%
				Fixed Income Arbitrage				
	31.3%	50.0%	43.8%	31.3%	37.5%	31.3%	25.0%	12.5%
	18.8%	6.3%	18.8%	18.8%	6.3%	6.3%	6.3%	0.0%
				MBS Arbitrage				
	20.0%	30.0%	30.0%	40.0%	30.0%	20.0%	10.0%	0.0%
	10.0%	0.0%	10.0%	20.0%	10.0%	0.0%	0.0%	0.0%

Table 3: Kupiec Test for Unconditional Coverage and Christoffersen Joint Test. The numbers in the first (second) row for each strategy refer to the 95% (99%) VaR prediction and represent the percentage of hedge funds for which the respective test is rejected at 5% significance level.

Merger Aribtrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
	1	0.0%	18.5%	3.7%	0.0%	3.7%	14.8%	22.2%	33.3%
	2	14.8%	22.2%	3.7%	11.1%	0.0%	33.3%	29.6%	7.4%
	3	14.8%	14.8%	7.4%	37.0%	7.4%	18.5%	25.9%	3.7%
	4	33.3%	7.4%	7.4%	33.3%	3.7%	14.8%	11.1%	0.0%
95%	5	18.5%	11.1%	18.5%	3.7%	7.4%	11.1%	3.7%	14.8%
	6	3.7%	7.4%	22.2%	3.7%	48.1%	3.7%	3.7%	7.4%
	7	3.7%	14.8%	22.2%	3.7%	18.5%	0.0%	0.0%	0.0%
	8	3.7%	0.0%	11.1%	0.0%	3.7%	0.0%	0.0%	0.0%
	Accurate VaR	**7.4%**	**0.0%**	**0.0%**	**7.4%**	**3.7%**	**0.0%**	**0.0%**	**33.3%**
99%		There are two many instances of accurate predictions across models. Therefore, meaningful comparison of deviation sizes across models cannot be made.							
	Accurate VaR	**59.3%**	**74.1%**	**81.5%**	**59.3%**	**85.2%**	**85.2%**	**88.9%**	**100.0%**

Convertible Bond Arbirage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
	1	0.0%	0.0%	0.0%	0.0%	5.9%	11.8%	11.8%	76.5%
	2	0.0%	0.0%	5.9%	0.0%	0.0%	35.3%	58.8%	11.8%
	3	23.5%	5.9%	5.9%	11.8%	17.6%	23.5%	23.5%	11.8%
	4	29.4%	23.5%	23.5%	47.1%	17.6%	17.6%	5.9%	0.0%
95%	5	41.2%	23.5%	11.8%	29.4%	5.9%	11.8%	0.0%	0.0%
	6	5.9%	23.5%	29.4%	11.8%	23.5%	0.0%	0.0%	0.0%
	7	0.0%	23.5%	11.8%	0.0%	23.5%	0.0%	0.0%	0.0%
	8	0.0%	0.0%	11.8%	0.0%	5.9%	0.0%	0.0%	0.0%
	Accurate VaR	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
	1	0.0%	11.8%	11.8%	0.0%	17.6%	17.6%	5.9%	47.1%
	2	11.8%	29.4%	23.5%	11.8%	17.6%	11.8%	0.0%	5.9%
	3	0.0%	23.5%	29.4%	0.0%	17.6%	5.9%	17.6%	23.5%
	4	58.8%	17.6%	5.9%	11.8%	5.9%	0.0%	0.0%	5.9%
99%	5	5.9%	11.8%	5.9%	23.5%	23.5%	11.8%	17.6%	0.0%
	6	5.9%	5.9%	17.6%	23.5%	11.8%	23.5%	17.6%	0.0%
	7	17.6%	0.0%	0.0%	11.8%	0.0%	23.5%	29.4%	0.0%
	8	0.0%	0.0%	5.9%	17.6%	5.9%	5.9%	11.8%	0.0%
	Accurate VaR	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**17.6%**

Table 4: Loss Function Metric for the Distance between Realized Losses and CVaR Forecasts, Conditional on VaR Exceedance, for the Merger Arbitrage and Convertible Bond Arbitrage Strategies. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the four panels.

Directional Credit

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
	1	5.0%	7.5%	2.5%	5.0%	0.0%	20.0%	17.5%	40.0%
	2	7.5%	10.0%	0.0%	15.0%	0.0%	17.5%	42.5%	2.5%
	3	15.0%	10.0%	0.0%	5.0%	0.0%	27.5%	27.5%	10.0%
	4	15.0%	20.0%	5.0%	32.5%	7.5%	12.5%	2.5%	0.0%
95%	5	30.0%	10.0%	2.5%	25.0%	5.0%	12.5%	5.0%	7.5%
	6	20.0%	27.5%	5.0%	7.5%	10.0%	5.0%	0.0%	22.5%
	7	2.5%	5.0%	65.0%	2.5%	17.5%	2.5%	0.0%	2.5%
	8	2.5%	5.0%	15.0%	2.5%	57.5%	0.0%	2.5%	2.5%
	Accurate VaR	**2.5%**	**5.0%**	**5.0%**	**5.0%**	**2.5%**	**2.5%**	**2.5%**	**12.5%**
	1	5.0%	50.0%	2.5%	2.5%	2.5%	7.5%	0.0%	22.5%
	2	40.0%	22.5%	10.0%	0.0%	5.0%	2.5%	5.0%	5.0%
	3	27.5%	7.5%	2.5%	2.5%	12.5%	2.5%	17.5%	7.5%
	4	10.0%	5.0%	25.0%	5.0%	15.0%	22.5%	5.0%	2.5%
99%	5	2.5%	0.0%	10.0%	27.5%	15.0%	10.0%	10.0%	0.0%
	6	0.0%	0.0%	10.0%	7.5%	5.0%	30.0%	22.5%	0.0%
	7	0.0%	0.0%	7.5%	17.5%	10.0%	12.5%	25.0%	0.0%
	8	0.0%	0.0%	2.5%	20.0%	2.5%	7.5%	5.0%	0.0%
	Accurate VaR	**15.0%**	**15.0%**	**30.0%**	**17.5%**	**32.5%**	**5.0%**	**10.0%**	**62.5%**

Long/Short Credit

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
	1	0.0%	5.9%	5.9%	0.0%	5.9%	11.8%	23.5%	47.1%
	2	5.9%	11.8%	11.8%	0.0%	0.0%	41.2%	17.6%	5.9%
	3	11.8%	5.9%	0.0%	11.8%	0.0%	23.5%	23.5%	17.6%
	4	11.8%	29.4%	5.9%	23.5%	11.8%	5.9%	5.9%	0.0%
95%	5	29.4%	17.6%	0.0%	29.4%	0.0%	0.0%	5.9%	5.9%
	6	17.6%	11.8%	23.5%	11.8%	17.6%	11.8%	0.0%	5.9%
	7	5.9%	5.9%	23.5%	11.8%	29.4%	0.0%	11.8%	0.0%
	8	11.8%	5.9%	23.5%	0.0%	29.4%	5.9%	0.0%	0.0%
	Accurate VaR	**5.9%**	**5.9%**	**5.9%**	**11.8%**	**5.9%**	**0.0%**	**11.8%**	**17.6%**
	1	5.9%	58.8%	5.9%	0.0%	0.0%	0.0%	0.0%	23.5%
	2	35.3%	5.9%	23.5%	0.0%	5.9%	5.9%	5.9%	5.9%
	3	23.5%	0.0%	23.5%	0.0%	29.4%	5.9%	5.9%	0.0%
	4	5.9%	11.8%	17.6%	11.8%	23.5%	5.9%	11.8%	0.0%
99%	5	11.8%	5.9%	5.9%	23.5%	11.8%	11.8%	5.9%	5.9%
	6	5.9%	0.0%	0.0%	5.9%	5.9%	41.2%	11.8%	0.0%
	7	0.0%	5.9%	5.9%	17.6%	0.0%	5.9%	35.3%	0.0%
	8	0.0%	0.0%	0.0%	23.5%	0.0%	5.9%	5.9%	0.0%
	Accurate VaR	**11.8%**	**11.8%**	**17.6%**	**17.6%**	**23.5%**	**17.6%**	**17.6%**	**64.7%**

Table 5: Loss Function Metric for the Distance between Realized Losses and CVaR Forecasts, Conditional on VaR Exceedance, for the Directional Credit and Long/Short Credit Strategies. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the four panels.

Fixed Income Arbitrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
95%	1	18.8%	6.3%	0.0%	0.0%	0.0%	25.0%	12.5%	37.5%
	2	0.0%	12.5%	0.0%	25.0%	6.3%	12.5%	37.5%	0.0%
	3	6.3%	12.5%	6.3%	0.0%	0.0%	31.3%	25.0%	6.3%
	4	18.8%	6.3%	0.0%	31.3%	18.8%	6.3%	6.3%	0.0%
	5	37.5%	12.5%	0.0%	18.8%	6.3%	6.3%	6.3%	0.0%
	6	0.0%	12.5%	25.0%	12.5%	18.8%	12.5%	0.0%	6.3%
	7	6.3%	18.8%	37.5%	0.0%	12.5%	6.3%	0.0%	6.3%
	8	0.0%	12.5%	18.8%	0.0%	25.0%	0.0%	0.0%	6.3%
	Accurate VaR	**12.5%**	**6.3%**	**12.5%**	**12.5%**	**12.5%**	**0.0%**	**12.5%**	**37.5%**
99%	1	12.5%	25.0%	6.3%	6.3%	0.0%	0.0%	12.5%	25.0%
	2	18.8%	12.5%	18.8%	6.3%	12.5%	12.5%	0.0%	6.3%
	3	12.5%	18.8%	12.5%	0.0%	6.3%	0.0%	6.3%	12.5%
	4	12.5%	18.8%	12.5%	0.0%	12.5%	6.3%	0.0%	0.0%
	5	12.5%	0.0%	12.5%	12.5%	25.0%	0.0%	6.3%	0.0%
	6	6.3%	0.0%	0.0%	25.0%	0.0%	18.8%	6.3%	6.3%
	7	0.0%	0.0%	0.0%	12.5%	12.5%	12.5%	25.0%	6.3%
	8	0.0%	0.0%	6.3%	12.5%	0.0%	18.8%	12.5%	0.0%
	Accurate VaR	**25.0%**	**25.0%**	**31.3%**	**25.0%**	**31.3%**	**31.3%**	**31.3%**	**43.8%**
					MBS Arbitrage				
95%	1	0.0%	0.0%	10.0%	0.0%	0.0%	10.0%	20.0%	40.0%
	2	0.0%	10.0%	0.0%	0.0%	30.0%	10.0%	40.0%	0.0%
	3	10.0%	10.0%	0.0%	0.0%	0.0%	30.0%	10.0%	20.0%
	4	20.0%	10.0%	10.0%	0.0%	10.0%	10.0%	10.0%	0.0%
	5	20.0%	20.0%	10.0%	20.0%	0.0%	0.0%	10.0%	0.0%
	6	10.0%	0.0%	10.0%	10.0%	20.0%	30.0%	0.0%	0.0%
	7	0.0%	10.0%	30.0%	30.0%	0.0%	0.0%	0.0%	10.0%
	8	10.0%	20.0%	10.0%	0.0%	20.0%	0.0%	0.0%	0.0%
	Accurate VaR	**30.0%**	**20.0%**	**20.0%**	**40.0%**	**20.0%**	**10.0%**	**10.0%**	**30.0%**
99%	1	10.0%	10.0%	10.0%	20.0%	10.0%	0.0%	0.0%	10.0%
	2	10.0%	20.0%	0.0%	0.0%	10.0%	10.0%	0.0%	20.0%
	3	20.0%	10.0%	20.0%	0.0%	0.0%	10.0%	10.0%	0.0%
	4	10.0%	10.0%	10.0%	0.0%	10.0%	10.0%	10.0%	0.0%
	5	0.0%	10.0%	0.0%	10.0%	10.0%	0.0%	10.0%	20.0%
	6	20.0%	0.0%	0.0%	0.0%	10.0%	10.0%	20.0%	0.0%
	7	0.0%	0.0%	10.0%	10.0%	10.0%	10.0%	20.0%	0.0%
	8	0.0%	0.0%	10.0%	20.0%	0.0%	20.0%	0.0%	0.0%
	Accurate VaR	**30.0%**	**40.0%**	**40.0%**	**40.0%**	**40.0%**	**30.0%**	**30.0%**	**50.0%**

Table 6: Loss Function Metric for the Distance between Realized Losses and CVaR Forecasts, Conditional on VaR Exceedance, for the Fixed Income Arbitrage and Mortgage-Backed Securities (MBS) Arbitrage Strategies. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the four panels.

Merger Arbitrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	0.0%	3.7%	0.0%	0.0%	0.0%	0.0%	0.0%	51.9%
95%	2	7.4%	29.6%	7.4%	0.0%	14.8%	33.3%	7.4%	0.0%
	3	29.6%	7.4%	11.1%	14.8%	3.7%	25.9%	18.5%	3.7%
	4	22.2%	7.4%	14.8%	25.9%	14.8%	3.7%	11.1%	0.0%
	Accurate VaR	**7.4%**	**18.5%**	**11.1%**	**7.4%**	**7.4%**	**7.4%**	**22.2%**	**44.4%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	11.1%	11.1%	11.1%	0.0%	25.9%	14.8%	7.4%	14.8%
95%	2	7.4%	11.1%	18.5%	14.8%	22.2%	11.1%	3.7%	3.7%
	3	3.7%	3.7%	18.5%	11.1%	7.4%	33.3%	7.4%	14.8%
	4	0.0%	18.5%	14.8%	11.1%	11.1%	22.2%	18.5%	0.0%
	Accurate VaR	**7.4%**	**18.5%**	**11.1%**	**7.4%**	**7.4%**	**7.4%**	**22.2%**	**44.4%**

Convertible Bond Arbitrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	5.9%	0.0%	0.0%	0.0%	5.9%	0.0%	0.0%	88.2%
95%	2	0.0%	29.4%	0.0%	5.9%	0.0%	35.3%	35.3%	5.9%
	3	11.8%	11.8%	29.4%	11.8%	23.5%	23.5%	29.4%	0.0%
	4	35.3%	23.5%	23.5%	29.4%	23.5%	11.8%	11.8%	0.0%
	Accurate VaR	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	0.0%	0.0%	0.0%	0.0%	17.6%	0.0%	0.0%	76.5%
95%	2	0.0%	0.0%	23.5%	0.0%	52.9%	17.6%	11.8%	11.8%
	3	0.0%	17.6%	23.5%	5.9%	11.8%	23.5%	23.5%	0.0%
	4	17.6%	5.9%	17.6%	17.6%	11.8%	23.5%	23.5%	0.0%
	Accurate VaR	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**

Table 7: Loss Function Metric for the Distance between Realized Losses and VaR Forecasts, Conditional on VaR Exceedance, for the Merger Arbitrage and Convertible Bond Arbitrage Strategies During the Month of September 2008. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the two panels. With space considerations in mind, results are provided only for the first four ranks and the 95% VaR. The complete results are available from the authors upon request.

Directional Credit

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	2.5%	5.0%	0.0%	0.0%	5.0%	2.5%	5.0%	65.0%
95%	2	7.5%	25.0%	15.0%	0.0%	2.5%	10.0%	22.5%	0.0%
	3	30.0%	2.5%	7.5%	7.5%	7.5%	12.5%	15.0%	0.0%
	4	17.5%	10.0%	7.5%	20.0%	15.0%	10.0%	2.5%	0.0%
	Accurate VaR	**20.0%**	**20.0%**	**17.5%**	**20.0%**	**17.5%**	**20.0%**	**17.5%**	**32.5%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	2.5%	12.5%	2.5%	10.0%	22.5%	2.5%	5.0%	27.5%
95%	2	5.0%	7.5%	15.0%	12.5%	22.5%	5.0%	7.5%	7.5%
	3	12.5%	10.0%	15.0%	5.0%	10.0%	17.5%	12.5%	0.0%
	4	7.5%	15.0%	7.5%	5.0%	0.0%	25.0%	17.5%	2.5%
	Accurate VaR	**20.0%**	**20.0%**	**17.5%**	**22.5%**	**17.5%**	**20.0%**	**17.5%**	**32.5%**
		Long/Short Credit							
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	0.0%	5.9%	0.0%	5.9%	5.9%	5.9%	0.0%	64.7%
95%	2	5.9%	17.6%	0.0%	5.9%	0.0%	23.5%	23.5%	0.0%
	3	5.9%	11.8%	17.6%	5.9%	0.0%	17.6%	23.5%	0.0%
	4	17.6%	17.6%	11.8%	11.8%	11.8%	5.9%	5.9%	0.0%
	Accurate VaR	**23.5%**	**17.6%**	**17.6%**	**11.8%**	**17.6%**	**23.5%**	**23.5%**	**35.3%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	0.0%	11.8%	17.6%	5.9%	17.6%	5.9%	0.0%	23.5%
95%	2	0.0%	5.9%	17.6%	17.6%	23.5%	5.9%	5.9%	5.9%
	3	11.8%	5.9%	17.6%	0.0%	11.8%	5.9%	17.6%	11.8%
	4	5.9%	11.8%	5.9%	0.0%	0.0%	23.5%	23.5%	11.8%
	Accurate VaR	**23.5%**	**17.6%**	**17.6%**	**17.6%**	**17.6%**	**23.5%**	**23.5%**	**35.3%**

Table 8: Loss Function Metric for the Distance between Realized Losses and VaR Forecasts, Conditional on VaR Exceedance, for the Directional Credit and Long/Short Credit Strategies During the Month of September 2008. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the two panels. With space considerations in mind, results are provided only for the first four ranks and the 95% VaR. The complete results are available from the authors upon request.

Fixed Income Aribtrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	0.0%	0.0%	0.0%	0.0%	6.3%	6.3%	12.5%	43.8%
95%	2	6.3%	0.0%	6.3%	6.3%	12.5%	31.3%	12.5%	0.0%
	3	12.5%	18.8%	0.0%	6.3%	6.3%	0.0%	31.3%	0.0%
	4	31.3%	12.5%	6.3%	6.3%	12.5%	0.0%	0.0%	0.0%
	Accurate VaR	**31.3%**	**25.0%**	**25.0%**	**31.3%**	**25.0%**	**31.3%**	**25.0%**	**56.3%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	12.5%	0.0%	0.0%	0.0%	18.8%	0.0%	18.8%	25.0%
95%	2	6.3%	0.0%	6.3%	12.5%	18.8%	25.0%	0.0%	6.3%
	3	0.0%	12.5%	18.8%	6.3%	12.5%	6.3%	18.8%	0.0%
	4	0.0%	12.5%	18.8%	12.5%	6.3%	6.3%	12.5%	6.3%
	Accurate VaR	**31.3%**	**25.0%**	**25.0%**	**31.3%**	**25.0%**	**31.3%**	**25.0%**	**56.3%**
MBS Arbitrage									
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	0.0%	0.0%	10.0%	0.0%	0.0%	0.0%	0.0%	40.0%
95%	2	10.0%	10.0%	0.0%	10.0%	0.0%	10.0%	10.0%	0.0%
	3	10.0%	0.0%	0.0%	10.0%	0.0%	10.0%	10.0%	0.0%
	4	20.0%	0.0%	0.0%	10.0%	10.0%	0.0%	0.0%	0.0%
	Accurate VaR	**60.0%**	**60.0%**	**50.0%**	**50.0%**	**60.0%**	**60.0%**	**60.0%**	**60.0%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	0.0%	0.0%	20.0%	0.0%	0.0%	0.0%	0.0%	30.0%
95%	2	0.0%	10.0%	0.0%	0.0%	20.0%	0.0%	10.0%	0.0%
	3	0.0%	0.0%	10.0%	0.0%	20.0%	10.0%	0.0%	0.0%
	4	0.0%	10.0%	0.0%	10.0%	0.0%	10.0%	10.0%	0.0%
	Accurate VaR	**60.0%**	**60.0%**	**50.0%**	**60.0%**	**60.0%**	**60.0%**	**60.0%**	**60.0%**

Table 9: Loss Function Metric for the Distance between Realized Losses and VaR Forecasts, Conditional on VaR Exceedance, for the Fixed Income Arbitrage and Mortgage-Backed Securities (MBS) Arbitrage During the Month of September 2008. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the two panels. With space considerations in mind, results are provided only for the first four ranks and the 95% VaR. The complete results are available from the authors upon request.

Merger Arbitrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	18.5%	11.1%	3.7%	0.0%	11.1%	7.4%	11.1%	3.7%
95%	2	11.1%	7.4%	3.7%	14.8%	3.7%	7.4%	11.1%	0.0%
	3	7.4%	18.5%	3.7%	11.1%	11.1%	7.4%	0.0%	0.0%
	4	7.4%	3.7%	33.3%	11.1%	0.0%	3.7%	0.0%	0.0%
	Accurate VaR	**40.7%**	**48.1%**	**40.7%**	**40.7%**	**37.0%**	**74.1%**	**77.8%**	**96.3%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	11.1%	11.1%	14.8%	3.7%	7.4%	7.4%	11.1%	0.0%
95%	2	7.4%	0.0%	3.7%	25.9%	14.8%	3.7%	0.0%	3.7%
	3	14.8%	14.8%	3.7%	3.7%	11.1%	7.4%	3.7%	0.0%
	4	0.0%	3.7%	22.2%	3.7%	14.8%	7.4%	7.4%	0.0%
	Accurate VaR	**40.7%**	**48.1%**	**40.7%**	**40.7%**	**37.0%**	**74.1%**	**77.8%**	**96.3%**

Convertible Bond Arbitrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	5.9%	0.0%	0.0%	0.0%	0.0%	5.9%	47.1%	35.3%
95%	2	0.0%	0.0%	5.9%	0.0%	0.0%	52.9%	35.3%	0.0%
	3	41.2%	5.9%	0.0%	29.4%	0.0%	29.4%	5.9%	0.0%
	4	35.3%	35.3%	11.8%	41.2%	0.0%	0.0%	0.0%	0.0%
	Accurate VaR	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**11.8%**	**11.8%**	**64.7%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	0.0%	0.0%	5.9%	0.0%	0.0%	47.1%	35.3%	11.8%
95%	2	0.0%	11.8%	0.0%	0.0%	11.8%	17.6%	47.1%	11.8%
	3	0.0%	23.5%	11.8%	0.0%	35.3%	17.6%	5.9%	0.0%
	4	0.0%	17.6%	29.4%	17.6%	29.4%	5.9%	0.0%	0.0%
	Accurate VaR	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**5.9%**	**11.8%**	**11.8%**	**64.7%**

Table 10: Loss Function Metric for the Distance between Realized Losses and VaR Forecasts, Conditional on VaR Exceedance, for the Merger Arbitrage and Convertible Bond Arbitrage Strategies During the Month of October 2008. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the two panels. With space considerations in mind, results are provided only for the first four ranks and the 95% VaR. The complete results are available from the authors upon request.

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Directional Credit									
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	10.0%	7.5%	2.5%	0.0%	2.5%	10.0%	27.5%	20.0%
95%	2	7.5%	2.5%	5.0%	7.5%	0.0%	30.0%	22.5%	0.0%
	3	20.0%	15.0%	10.0%	10.0%	0.0%	12.5%	10.0%	0.0%
	4	12.5%	25.0%	7.5%	20.0%	12.5%	0.0%	0.0%	0.0%
	Accurate VaR	**22.5%**	**25.0%**	**22.5%**	**22.5%**	**20.0%**	**42.5%**	**40.0%**	**80.0%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	7.5%	5.0%	12.5%	0.0%	12.5%	12.5%	22.5%	7.5%
95%	2	0.0%	0.0%	15.0%	10.0%	15.0%	20.0%	17.5%	0.0%
	3	2.5%	17.5%	7.5%	0.0%	20.0%	17.5%	10.0%	2.5%
	4	0.0%	2.5%	20.0%	17.5%	22.5%	7.5%	7.5%	0.0%
	Accurate VaR	**22.5%**	**25.0%**	**22.5%**	**22.5%**	**20.0%**	**42.5%**	**40.0%**	**80.0%**
Long/Short Credit									
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	11.8%	0.0%	0.0%	5.9%	5.9%	23.5%	0.0%	35.3%
95%	2	11.8%	17.6%	0.0%	17.6%	0.0%	5.9%	23.5%	0.0%
	3	0.0%	5.9%	23.5%	11.8%	11.8%	17.6%	5.9%	0.0%
	4	23.5%	41.2%	5.9%	0.0%	5.9%	0.0%	0.0%	0.0%
	Accurate VaR	**29.4%**	**23.5%**	**23.5%**	**17.6%**	**23.5%**	**41.2%**	**58.8%**	**64.7%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	5.9%	11.8%	11.8%	5.9%	11.8%	5.9%	5.9%	17.6%
95%	2	5.9%	11.8%	5.9%	0.0%	23.5%	17.6%	11.8%	0.0%
	3	0.0%	11.8%	17.6%	17.6%	5.9%	17.6%	5.9%	0.0%
	4	5.9%	11.8%	17.6%	5.9%	17.6%	5.9%	11.8%	0.0%
	Accurate VaR	**29.4%**	**23.5%**	**23.5%**	**23.5%**	**23.5%**	**41.2%**	**58.8%**	**64.7%**

Table 11: Loss Function Metric for the Distance between Realized Losses and VaR Forecasts, Conditional on VaR Exceedance, for the Directional Credit and Long/Short Credit Strategies During the Month of October 2008. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the two panels. With space considerations in mind, results are provided only for the first four ranks and the 95% VaR. The complete results are available from the authors upon request.

Fixed Income Aribtrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	6.3%	0.0%	0.0%	0.0%	0.0%	6.3%	25.0%	18.8%
95%	2	6.3%	0.0%	6.3%	0.0%	0.0%	18.8%	18.8%	6.3%
	3	18.8%	0.0%	0.0%	6.3%	6.3%	18.8%	6.3%	0.0%
	4	18.8%	6.3%	6.3%	18.8%	6.3%	0.0%	0.0%	0.0%
	Accurate VaR	**43.8%**	**43.8%**	**43.8%**	**43.8%**	**43.8%**	**56.3%**	**50.0%**	**75.0%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	6.3%	6.3%	0.0%	0.0%	0.0%	12.5%	6.3%	25.0%
95%	2	0.0%	0.0%	6.3%	6.3%	12.5%	12.5%	18.8%	0.0%
	3	6.3%	6.3%	6.3%	0.0%	12.5%	12.5%	12.5%	0.0%
	4	0.0%	6.3%	12.5%	6.3%	18.8%	0.0%	12.5%	0.0%
	Accurate VaR	**43.8%**	**43.8%**	**43.8%**	**43.8%**	**43.8%**	**56.3%**	**50.0%**	**75.0%**

MBS Arbitrage

VaR/CVaR Level	Model Rank	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
		Metric: Distance between predicted VaR and realized loss, conditional on VaR exceedance							
	1	0.0%	0.0%	0.0%	0.0%	0.0%	10.0%	20.0%	20.0%
95%	2	10.0%	10.0%	0.0%	0.0%	0.0%	10.0%	20.0%	0.0%
	3	10.0%	0.0%	0.0%	10.0%	10.0%	20.0%	0.0%	0.0%
	4	10.0%	10.0%	20.0%	10.0%	0.0%	0.0%	0.0%	0.0%
	Accurate VaR	**60.0%**	**50.0%**	**50.0%**	**60.0%**	**50.0%**	**50.0%**	**50.0%**	**80.0%**
		Metric: Distance between predicted CVaR and realized loss, conditional on VaR exceedance							
	1	0.0%	0.0%	0.0%	0.0%	10.0%	0.0%	20.0%	20.0%
95%	2	0.0%	0.0%	20.0%	0.0%	0.0%	20.0%	10.0%	0.0%
	3	0.0%	10.0%	0.0%	0.0%	20.0%	20.0%	0.0%	0.0%
	4	0.0%	10.0%	20.0%	0.0%	10.0%	0.0%	10.0%	0.0%
	Accurate VaR	**60.0%**	**50.0%**	**50.0%**	**60.0%**	**50.0%**	**50.0%**	**50.0%**	**80.0%**

Table 12: Loss Function Metric for the Distance between Realized Losses and VaR Forecasts, Conditional on VaR Exceedance, for the Fixed Income Arbitrage and Mortgage-Backed Securities (MBS) Arbitrage Strategies During the Month of October 2008. The numbers are percentages of hedge funds for which the respective model is assigned a given rank. Higher rank, coupled with a greater percentage of hedge funds for which that rank is achieved, indicates superior model performance. Ranking should be analyzed in conjunction with the accuracy of VaR prediction. The percentage of hedge funds for which VaR is predicted accurately is given in the last rows of the two panels. With space considerations in mind, results are provided only for the first four ranks and the 95% VaR. The complete results are available from the authors upon request.

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